Practice 2: Digital Systems

Boolean Algebra:

A few important identities:

1. $A \cdot \overline{A} = 0 \quad A \cdot A = A = A + A$
2. $A + \overline{A} = 1 = 1 + x$
3. $A + \overline{A}B = A + B \quad \overline{A} + AB = \overline{A} + B$
4. $\overline{AB} = \overline{A} + \overline{B} \quad \overline{A + B} = \overline{A} \cdot \overline{B}$

Example 1: Function Minimization

Minimize the following expression:

$$\left(\overline{A} + AB\overline{C}\right) + \left(A + \overline{A}\overline{B}C\right)\left(\overline{A\left(\overline{A} + \overline{B} + C\right)}\right)$$

Solution:

$$\left(\overline{A} + AB\overline{C}\right) + \left(A + \overline{A}\overline{B}C\right)\left(\overline{A\left(\overline{A} + \overline{B} + C\right)}\right) =$$

$$\overline{A\left(\overline{A} + \overline{B} + C\right)} = \overline{A} + \overline{\overline{A} + \overline{B} + C} = \overline{A} + AB\overline{C} = \overline{A} + B\overline{C}$$

$$\left(\overline{A} + B\overline{C}\right) + \left(A + \overline{B}C\right)\left(\overline{A} + B\overline{C}\right) =$$

$$\left(\overline{A} + B\overline{C}\right)\left(1 + \left(A + \overline{B}C\right)\right) = \overline{A} + B\overline{C}$$

Example 2: Complementary Functions

Find the complementary expression for:

$$f = A + BC + \bar{A}\bar{B}$$

Solution:

$$\bar{f} = \overline{A + BC + \bar{A}\bar{B}} = (DeMorgan) =$$

$$= \bar{A} \cdot \overline{BC} \cdot \overline{\bar{A}\bar{B}} = \bar{A} \cdot (\bar{B} + \bar{C}) \cdot (A + B) =$$

$$\bar{A} \cdot (A + B) = \bar{A}B$$

$$= \bar{A}B \cdot (\bar{B} + \bar{C}) =$$

$$B \cdot (\bar{B} + \bar{C}) = B\bar{C}$$

$$= \bar{A}B\bar{C}$$

Example 3: Universality

Implement the following function with NAND and NOT gates:

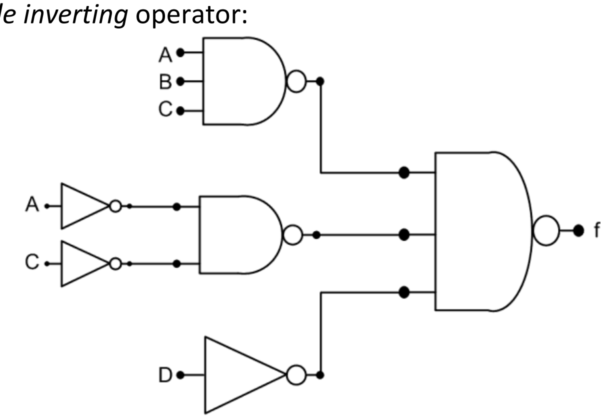
$$f = ABC + \bar{A}\bar{C} + D$$

Solution:

An important step in implementation of digital circuits is finding the *inverting* representation of the function. To do this we will apply a *double inverting* operator:

$$\bar{\bar{f}} = \overline{\overline{ABC + \bar{A}\bar{C} + D}} = (DeMorgan) =$$

$$= \overline{\overline{ABC} \cdot \overline{\bar{A}\bar{C}} \cdot \bar{D}}$$

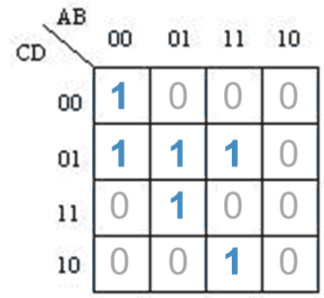


Example 4: Multiplexers

A Multiplexer is a complex gate that receives 2^N inputs, N selectors and propagates one output. Muxes are one of the most important and useful gates in digital design. In this example, we will implement a standard logic function using the universality of multiplexers:

Implement the following function using 4:1 multiplexers:

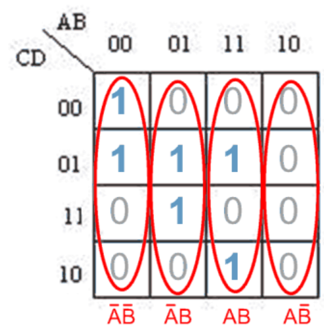
$$f(A,B,C,D)=\sum(0,1,5,7,13,14)$$



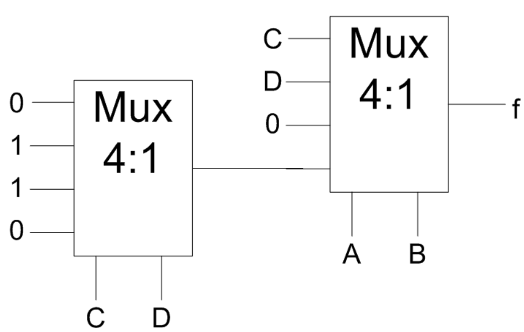
Solution:

1. First, we'll draw the Karnaugh Map of the function:
2. A 4:1 Multiplexer receives 2 selectors, so we can make two-variable circles to define the output of each state of the mux. We will choose AB as the selectors for the first mux:
3. As we can see, we have arrived at four partial expressions:

$$f(\bar{A}\bar{B})=\bar{C}$$
$$f(\bar{A}B)=D$$
$$f(AB)=C\oplus D$$
$$f(A\bar{B})=0$$



4. We can implement this function with two 4:1 muxes:



Bonus Question (if we have time)

A. Create a leading zeros detector for a four bit vector.

B. Use the answer to A to create a leading zeros detector for a 16-bit vector.

Bonus 2:

Using 3 Shift Registers and an n-bit adder, design a circuit that receives incrementing integers and outputs their squared value. (1, 4, 9, 16…)