Practice 10:

Ratioed Logic

Ratioed vs. Non-Ratioed

- Standard CMOS is a *non-Ratioed* logic family, because:
  - The logic function will be correctly implemented regardless of device sizing.
  - Device sizing will only affect the performance of the gate.

- A Ratioed gate is a circuit:
  - That will only function properly if a certain ratio is maintained between the drive strengths of its components.
  - If the required ratio is not met, the gate's output may be incorrect, the noise margins may become negative, or it may lose its regenerative property.

Why use ratioed circuits?

- Historically, before CMOS technology was available, it was hard to implement non-ratioed logic.

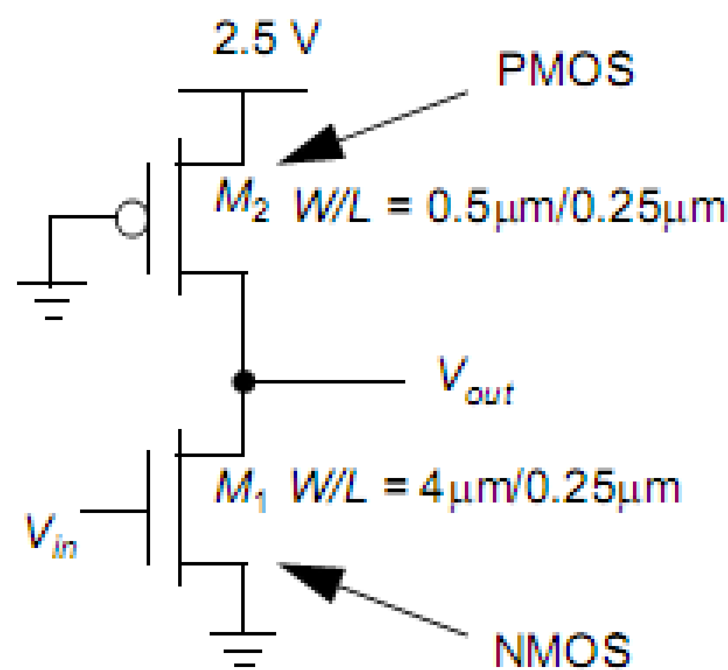
- Today, many techniques require ratioed circuits
  - Such as SRAM design.
- Pseudo-nMOS is a ratioed logic family that requires less transistors than CMOS and can optimize one transition.

Exercise 1: Pseudo nMOS

Exercise 1a

- For a pseudo-nMOS inverter with: $V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$
  - A. Draw the gate's VTC and compute noise margins.

$k_n' = 115\mu \quad k_p' = 30\mu \quad \lambda_p = \lambda_n = 0.1$



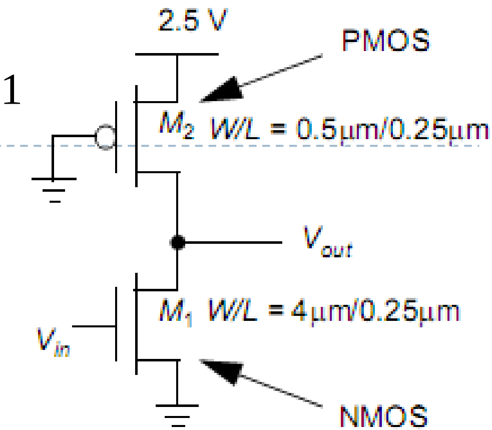
- Starting with Vin=0:
  - M1 is cut-off
  - M2 is in linear

$$V_{OH\max} = V_{DD} = 2.5V$$

Exercise 1a

$$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$$

$$k_n^{'} = 115\mu \quad k_p^{'} = 30\mu \quad \lambda_p = \lambda_n = 0.1$$



- Raising Vin, M1 turns on in Saturation, while M2 is still linear:

$$I_{SDp}(lin) = I_{DSn}(sat)$$

$$k_p^{'} \frac{W_p}{L_p}\left[\left(V_{SGp} - |V_{Tp}|\right)V_{SDp} - 0.5V_{SDp}^2\right] = k_n^{'} \frac{W_n}{2L_n}(V_{GSn} - V_{Tn})^2(1 + \lambda_n V_{DSn})$$

$$k_p\left[\left(V_{DD} - |V_{Tp}|\right)(V_{DD} - V_{out}) - 0.5(V_{DD} - V_{out})^2\right] = r \cdot k_p (V_{in} - V_{Tn})^2$$

$$V_{out} = V_T + \sqrt{(V_{DD} - V_{in})^2 - r(V_T - V_{in})^2}$$

$$k_n = rk_p$$

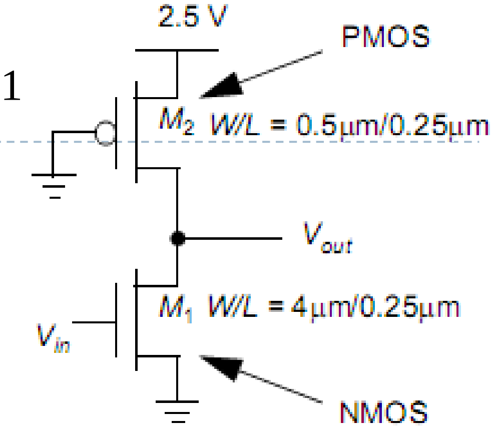
$$\left.\frac{dV_{out}}{dV_{in}}\right|_{V_{in}=V_{IL}} = -1 = \frac{d}{dV_{in}}\left[V_T + \sqrt{(V_{DD} - V_{in})^2 - r(V_T - V_{in})^2}\right]$$

$$V_{IL} = V_T + \frac{V_{DD} - V_T}{\sqrt{r(r+1)}}$$

Exercise 1a

$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$

$k_n^{'} = 115\mu \quad k_p^{'} = 30\mu \quad \lambda_p = \lambda_n = 0.1$



$$V_{IL} = V_T + \frac{V_{DD} - V_T}{\sqrt{r(r+1)}}$$

$$k_n = rk_p$$

- Substituting values gives us:

$$k_p = 30\mu \cdot \frac{0.5\mu}{0.25\mu} = 60\mu \qquad k_n = 115\mu \cdot \frac{4\mu}{0.25\mu} = 1840\mu \qquad r = \frac{1840\mu}{60\mu} = 30.666$$

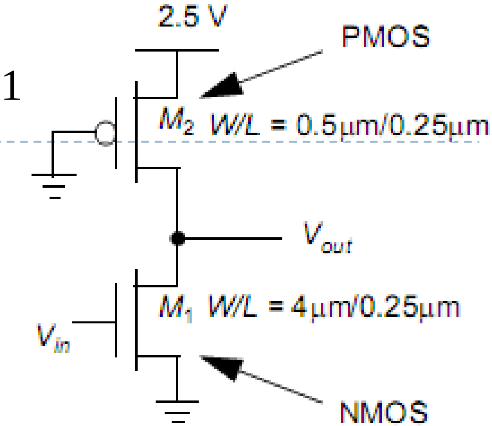
$$V_{IL} = V_T + \frac{V_{DD} - V_T}{\sqrt{r(r+1)}} = 0.4 + \frac{2.5 - 0.4}{\sqrt{971.11}} = 0.467V$$

$$V_{OH\min} = 0.4 + \sqrt{(2.5 - 0.467)^2 - 30.666(0.467 - 0.4)^2} = 2.4V$$

Exercise 1a

$$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$$

$$k_n' = 115\mu \quad k_p' = 30\mu \quad \lambda_p = \lambda_n = 0.1$$



- As Vin rises, Vout drops, causing M1 to enter linear and M2 to saturate:

$$I_{SDp}(sat) = I_{DSn}(lin)$$

$$k_n = rk_p$$

$$k_p' \frac{W_p}{2L_p}\left(V_{SGp} - |V_{Tp}|\right)^2 \left(1 + \lambda_p V_{SDp}\right) = k_n' \frac{W_n}{L_n}\left[\left(V_{GSn} - V_{Tn}\right)V_{DSn} - 0.5V_{DSn}^2\right]$$

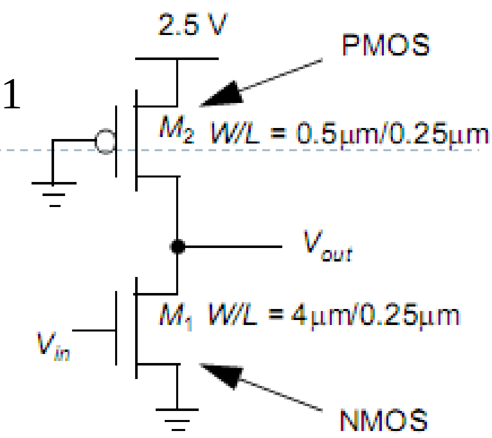
$$k_p\left(V_{DD} - |V_T|\right)^2 = r \cdot k_p\left[\left(V_{in} - V_T\right)V_{out} - 0.5V_{out}^2\right]$$

$$V_{out} = \left(V_{in} - V_T\right) - \sqrt{\left(V_{in} - V_T\right)^2 - \frac{1}{r}\left(V_{DD} - V_T\right)^2}$$

Exercise 1a

$$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$$

$$k_n^{'} = 115\mu \quad k_p^{'} = 30\mu \quad \lambda_p = \lambda_n = 0.1$$



$$V_{out} = (V_{in} - V_T) - \sqrt{(V_{in} - V_T)^2 - \frac{1}{r}(V_{DD} - V_T)^2}$$

- Differentiating, we find:

$$\left.\frac{dV_{out}}{dV_{in}}\right|_{V_{in}=V_{IH}} = -1 = \frac{d}{dV_{in}}\left[(V_{in} - V_T) - \sqrt{(V_{in} - V_T)^2 - \frac{1}{r}(V_{DD} - V_T)^2}\right]$$

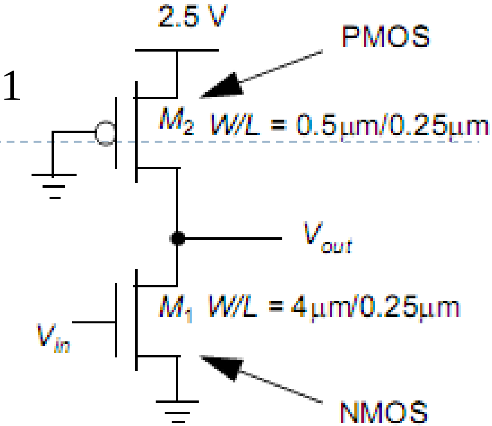
$$V_{IH} = V_T + \frac{2V_{DD} - V_T}{\sqrt{3r}} = 0.4 + \frac{5 - 0.4}{\sqrt{92}} = 0.88V$$

$$V_{OL\max} = (0.88 - 0.4) - \sqrt{(0.88 - 0.4)^2 - 0.0326(2.5 - 0.4)^2} = 0.18V$$

Exercise 1a

$$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$$

$$k_n^{'} = 115\mu \quad k_p^{'} = 30\mu \quad \lambda_p = \lambda_n = 0.1$$



- Finally, we will find V_{OLmin} by setting $V_{in}=V_{DD}=2.5V$
  - M1 is still in linear and M2 is still saturated
  - The equation we found before is still relevant:

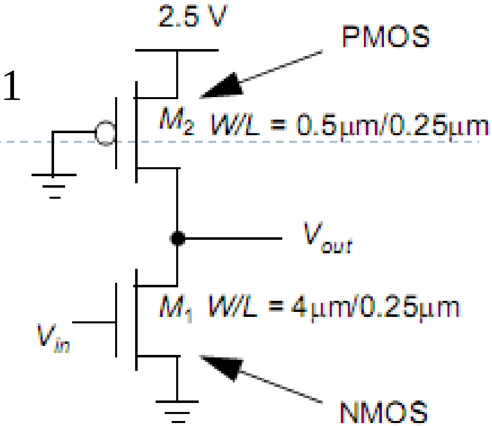
$$V_{out} = (V_{in} - V_T) - \sqrt{(V_{in} - V_T)^2 - \frac{1}{r}(V_{DD} - V_T)^2}$$

$$V_{OL\min}\Big|_{V_{in}=V_{DD}} = (V_{DD} - V_T)\left[1 - \sqrt{1 - \frac{1}{r}}\right] = (2.5 - 0.4)\left(1 - \sqrt{0.9674}\right) = 0.0345V$$

Exercise 1a

$$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$$

$$k_n' = 115\mu \quad k_p' = 30\mu \quad \lambda_p = \lambda_n = 0.1$$



- Now we can draw the VTC and find the Noise Margins

$$NM_H = V_{OH\,\min} - V_{IH} = 2.4 - 0.88 = 1.55V$$

$$NM_L = V_{IL} - V_{OL\,\max} = 0.467 - 0.18 = 0.28V$$

$$V_{OH\,\max} = 2.5V$$

$$V_{IL} = 0.467V$$

$$V_{OH\,\min} = 2.4V$$

$$V_{IH} = 0.88V$$

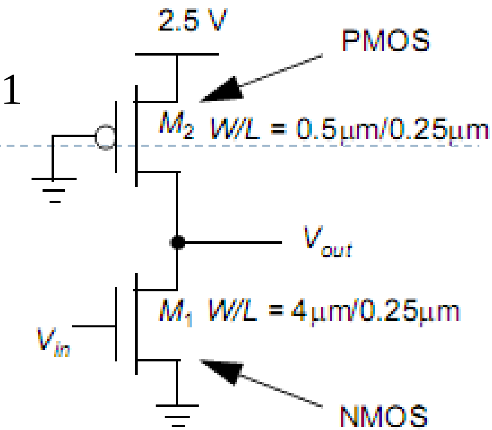
$$V_{OL\,\max} = 0.18$$

$$V_{OL\,\min} = 0.0345V$$

Exercise 1b

$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$

$k_n^{'} = 115\mu \quad k_p^{'} = 30\mu \quad \lambda_p = \lambda_n = 0.1$



- B. Find the power dissipation with high and low inputs. How is this different than CMOS?

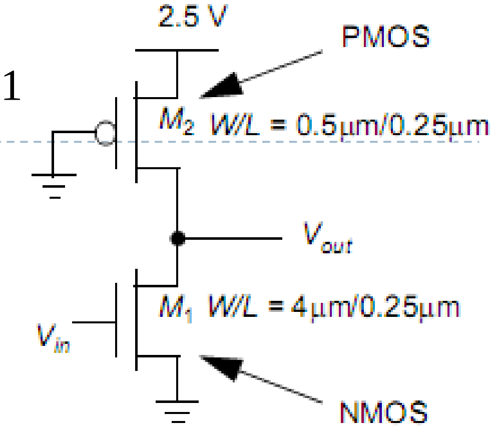
$$I_{static} = \frac{1}{2} k_p \left(V_{DD} - V_{Tp}\right)^2 \left(1 + \lambda\left(V_{DD} - V_{OL\min}\right)\right) = 165\mu A$$

$$P_{AV} = I_{static} \cdot V_{DD} = 412.5\mu W$$

Exercise 1c

$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$

$k_n^{'} = 115\mu \quad k_p^{'} = 30\mu \quad \lambda_p = \lambda_n = 0.1$

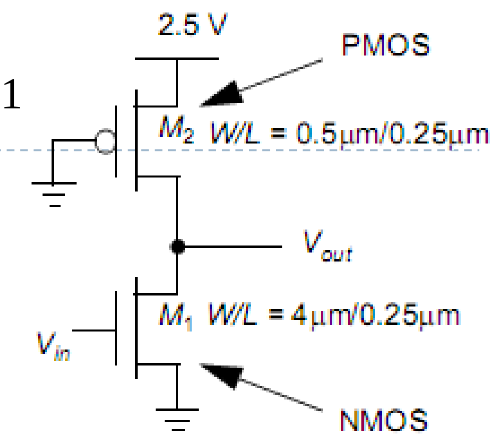


- C. Find the high-to-low propagation delay of the gate, with an ideal step at the input.
  - Assume a 10pF load is connected.
  - Use the average current approximation, but differentiate between currents in different operating modes.
- At t<0,
  - Vin=0,
  - Vout=V_{OHmax}=V_{DD}
- At t=0, Vin→VDD, and the output starts to discharge:
  - M1 – saturation
  - M2 – linear
- This continues until:
  - $V_{out}=V_{SGP}-V_T=V_{DD}-V_T$

Exercise 1c

$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$

$k_n^{'} = 115\mu \quad k_p^{'} = 30\mu \quad \lambda_p = \lambda_n = 0.1$



- Therefore we will find the current at:
  - $t=0 \rightarrow M1{:}sat,\ M2{:}lin$
  - $t=t_1\ (V_{out}=V_{DD}-V_T) \rightarrow M1{:}sat,\ M2{:}lin$
  - $t=t_{pd}\ (V_{out}=V_{DD}/2) \rightarrow M1{:}lin,\ M2{:}\ lin$

- t=0

$$V_{GTn} = V_{DD} - V_T \quad V_{DSn} = V_{DD} \quad \Rightarrow SAT$$

$$I_{DSn} = \frac{k_n}{2}(V_{DD} - V_T)^2(1+\lambda V_{DD}) = 920\mu(2.1)^2(1+\lambda 2.5) = 5mA$$

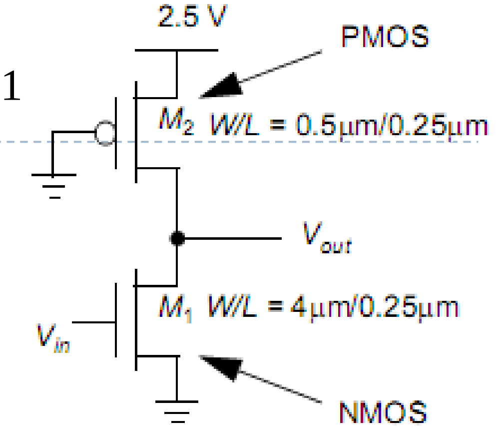
$$V_{GTp} = V_{DD} - V_T \quad V_{DSp} = 0 \quad \Rightarrow LIN$$

$$I_{SDp} = 0$$

Exercise 1c

$$V_{Tn}=|V_{Tp}|=0.4V \quad V_{DSAT}=V_{DD}$$

$$k_n^{'}=115\mu \quad k_p^{'}=30\mu \quad \lambda_p=\lambda_n=0.1$$



- **t=t$_l$**

$$\left.\begin{array}{l} V_{GTn}=V_{DD}-V_T \\ V_{DSn}=V_{DD}-V_T \end{array}\right. \Rightarrow pinch\,off$$

$$I_{DSn}=\frac{k_n}{2}(V_{DD}-V_T)^2\left(1+\lambda(V_{DD}-V_T)\right)=920\mu(2.1)^2(1+\lambda 2.1)=4.9mA$$

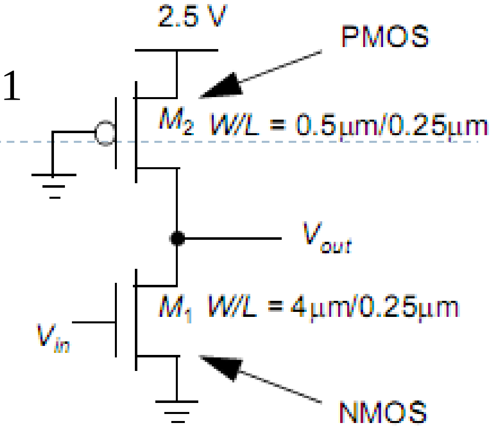
$$V_{GTp}=V_{DD}-V_T \quad V_{DSp}=V_T \Rightarrow LIN$$

$$I_{SDp}=k_p\left[(V_{DD}-V_T)V_T-0.5V_T^2\right]=60\mu\left[(2.1)0.4-0.5\cdot 0.4^2\right]=45.6\mu A$$

Exercise 1c

$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$

$k_n' = 115\mu \quad k_p' = 30\mu \quad \lambda_p = \lambda_n = 0.1$



- **$t = t_{pd}$**

$$\left.\begin{aligned} V_{GTn} &= V_{DD} - V_T \\ V_{DSn} &= V_{DD}/2 \end{aligned}\right\} \Rightarrow LIN$$

$$I_{DSn} = k_n\left[(V_{DD} - V_T)\frac{V_{DD}}{2} - 0.5\left(\frac{V_{DD}}{2}\right)^2\right] = 1840\mu\left[(2.1)1.25 - 0.5\cdot 1.25^2\right] = 3.4mA$$

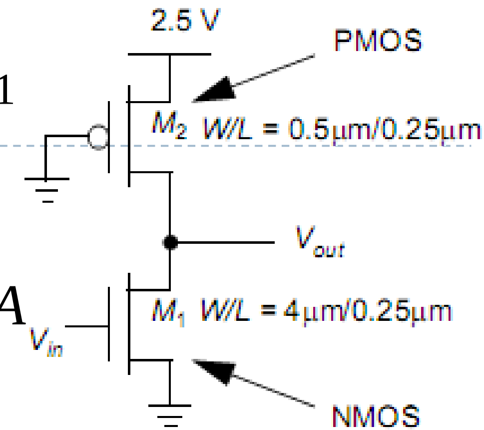
$$\left.\begin{aligned} V_{GTp} &= V_{DD} - V_T \\ V_{DSp} &= V_{DD}/2 \end{aligned}\right\} \Rightarrow LIN$$

$$I_{SDp} = k_p\left[(V_{DD} - V_T)\frac{V_{DD}}{2} - 0.5\left(\frac{V_{DD}}{2}\right)^2\right] = 60\mu\left[(2.1)1.25 - 0.5\cdot 1.25^2\right] = 110.6\mu A$$

Exercise 1c

$$V_{Tn} = |V_{Tp}| = 0.4V \quad V_{DSAT} = V_{DD}$$

$$k_n' = 115\mu \quad k_p' = 30\mu \quad \lambda_p = \lambda_n = 0.1$$



$$I_{t_0} = I_{n0} = 5mA \quad I_{t_1} = I_{n1} - I_{p1} = 4.9m - 45.6\mu = 4.85mA$$

$$I_{tpd} = I_{tpd,n} - I_{tpd,p} = 3.4m - 110.6\mu = 3.29mA$$

- Calculate t_1-0:

$$(t_1 - 0) = C_L \frac{V_{DD} - (V_{DD} - V_T)}{0.5(I_{t_o} + I_{t_1})} = \frac{10p \cdot 0.4}{4.925m} = 812ps$$

- Calculate t_{pd}-t_1:

$$(t_{pd} - t_1) = C_L \frac{(V_{DD} - V_T) - V_{DD}/2}{0.5(I_{t_1} + I_{t_{pd}})} = \frac{10p \cdot 0.85}{4.07m} = 2ns$$