Practice 1: RC Circuits

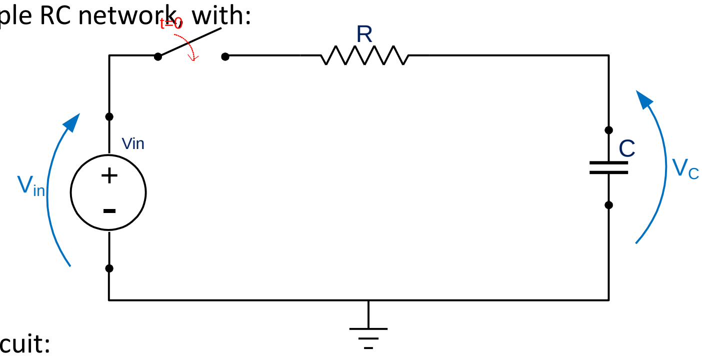
Example 1:

Find the output of the following simple RC network, with:

R, C, Vin given.

The switch closes at t=0.

$V_c(0)=V_0$



Solution:

1. Let's write the KVL of the circuit:

$$V_{in}(t)=V_c(t)+V_R(t)=V_c(t)+i_C(t)R$$

$$V_{in}=V_c+RC\dot{V}_c=\tau\dot{V}_c+V_c \qquad \tau\triangleq RC$$

2. Next we'll find the homogenous solution to the simple differential equation:

$$V_{ch}(t)=Ke^{-t/\tau}$$

3. The solution is the homogenous plus a particular solution, so we'll guess the particular solution:

$$V_{cp}(t)=V_{in}$$

4. And find K, using our knowledge that $V_c(0)=V_0$:

$$V_c(t)=V_{ch}+V_{cp}=Ke^{-t/\tau}+V_{in}$$

$$V_c(0)=V_0=Ke^{-0/\tau}+V_{in}=K+V_{in}$$

$$K=V_0-V_{in}$$

5. Finally, we'll put it all together:

$$V_c(t)=(V_0-V_{in})e^{-t/RC}+V_{in}=V_0e^{-t/RC}+\left(1-e^{-t/RC}\right)V_{in}$$

6. It is useful to extract *t* from this equation, in order to find time delays depending on voltages:

$$e^{-t/RC}=\frac{V_c-V_{in}}{V_0-V_{in}}$$

$$t=RC\ln\frac{V_0-V_{in}}{V_c-V_{in}}\equiv\tau\ln\frac{V_c(0)-V_c(\infty)}{V_c(t)-V_c(\infty)}$$

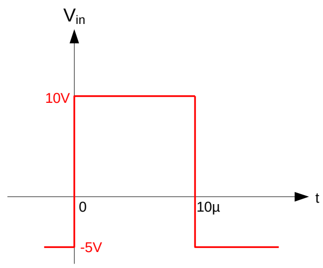
Exercise 2:

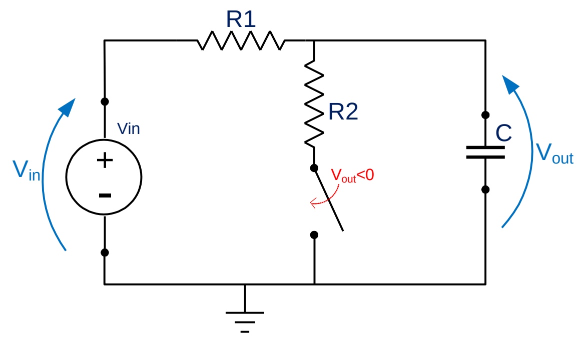
The following circuit is given with:

R1=100KΩ, R2=2KΩ, C=100pF

The switch is closed when $V_{out}<0$

Vin:

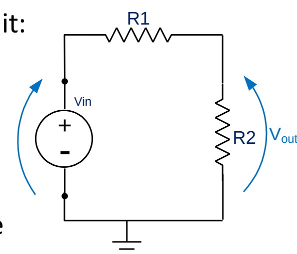




Draw a graph of $V_{out}(t)$

Solution:

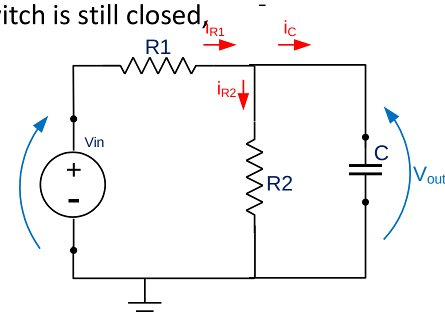
1. We'll start at t<0. We have V_{in}=-5V for a long time (steady state), so the capacitor is an open circuit. Since the input voltage is negative, the voltage on the resistors is negative and the switch is closed. We get the following equivalent circuit:



$$V_{in} = -5V = V_{R1} + V_{out}$$

$$V_{out} = V_{in} \cdot \frac{R_2}{R_1 + R_2} = -5 \cdot \frac{2K}{102K} \approx -0.1V$$

2. At t=0, the voltage changes to V_{in}=10V. The capacitor's voltage stays continuous, so $V_c(0^-)= V_c(0^-)=V_{out}(t<0)$. The switch is still closed, so our circuit is now:



$$i_{R1} = i_{R2} + i_C$$

$$\frac{V_{in} - V_{out}}{R1} = \frac{V_{out}}{R2} + C\dot{V}_{out}$$

$$C\dot{V}_{out} + \frac{R1 + R2}{R1 \cdot R2} V_{out} = \frac{V_{in}}{R1}$$

$$C\frac{R1 \cdot R2}{R1 + R2}\dot{V}_{out} + V_{out} = \frac{R2}{R1 + R2} V_{in}$$

$$\tau_1 \dot{V}_{out} + V_{out} = V_\infty \qquad \tau_1 = \frac{C \cdot R1 \cdot R2}{R1 + R2} = 0.196\mu\text{sec} \qquad V_\infty = \frac{R2}{R1 + R2} V_{in} = 0.2V$$

3. From the first exercise, we can find the time that the switch opens (V_{out}=0):

$$t_1 = \tau_1 \ln \frac{V_0 - V_\infty}{V_{out}(t_1) - V_\infty} = 0.2\mu \cdot \ln \frac{-0.2 - 0.1}{0 - 0.2} = 0.08\mu\text{sec}$$

4. Now the switch opens and we get a standard discharging RC network until t_2=10μsec:

$$V_c(t-t_1) = (V_0 - V_{in})e^{-t/R1\cdot C} + V_{in}$$

$$V_c(10\mu - 0.08\mu) = (0-10)e^{-9.92\mu/10\mu} + 10 = -10e^{-0.992} + 10 = 6.29V$$

5. At t=10μ, the Vin=-5V. The capacitor again has voltage continuity and the switch stays closed. We have the same RC network with a different V_{in}. We will now find the time when the switch opens again:

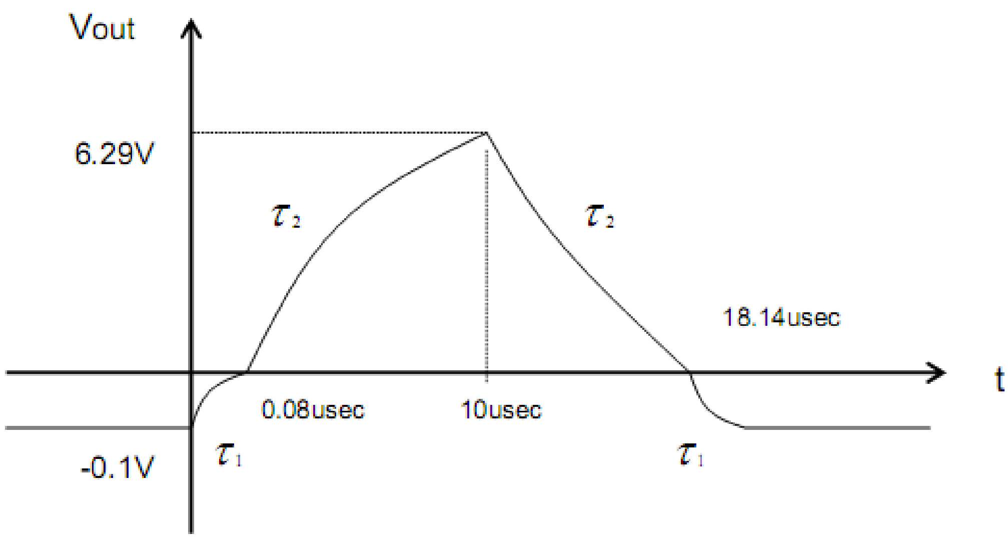
$$t_3 - t_2 = RC\ln\frac{V_{out}(t_2) - V_{in}}{V_{out}(t_3) - V_{in}} = 10\mu \cdot \ln\frac{6.29+5}{0+5} = 8.14\mu\sec$$

$$t_3 = t_2 + 8.14\mu = 18.14\mu\sec$$

6. The switch now closes again so we return to our second equivalent circuit, though now Vin=-5V:

$$\tau_1 \overset{\circ}{V}_{out}(t-t_3) + V_{out}(t-t_3) = V_\infty \qquad V_{out}(0) = 0$$

$$\tau_1 = 0.196\mu\sec \quad V_\infty = \frac{R2\cdot V_{in}}{R1+R2} = -0.098V$$



$$\tau_1 = 0.2[\mu Sec]$$

$$\tau_2 = 10.0[\mu Sec]$$