Digital Integrated Circuits
(83-313)

Lecture 11:
MOSFET Modeling

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MOSFET Current Modeling

- In Digital Electronic Circuits, we used the *Shockley Model* for hand-analysis of circuit operation.
- However, there are many models with varying levels of accuracy to estimate the I-V curves of a MOSFET.
- In this section, we will overview several models that will come in use throughout this course and your future.

Lecture Content

Basic MOS Models

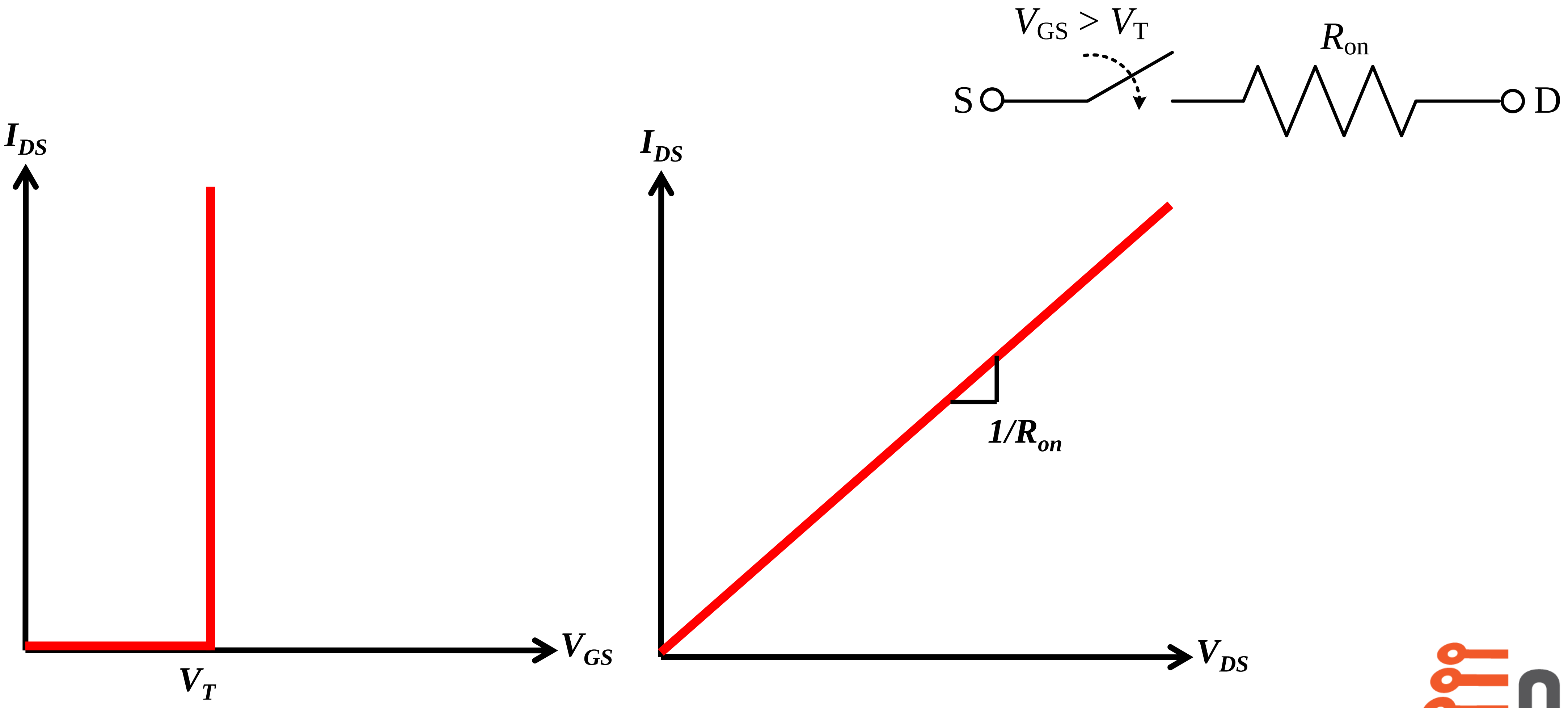
Advanced MOS Models

Threshold Voltage Revisited

Basic MOS Models

Switch Model

- **The most simple MOSFET model is the Switch Model.**

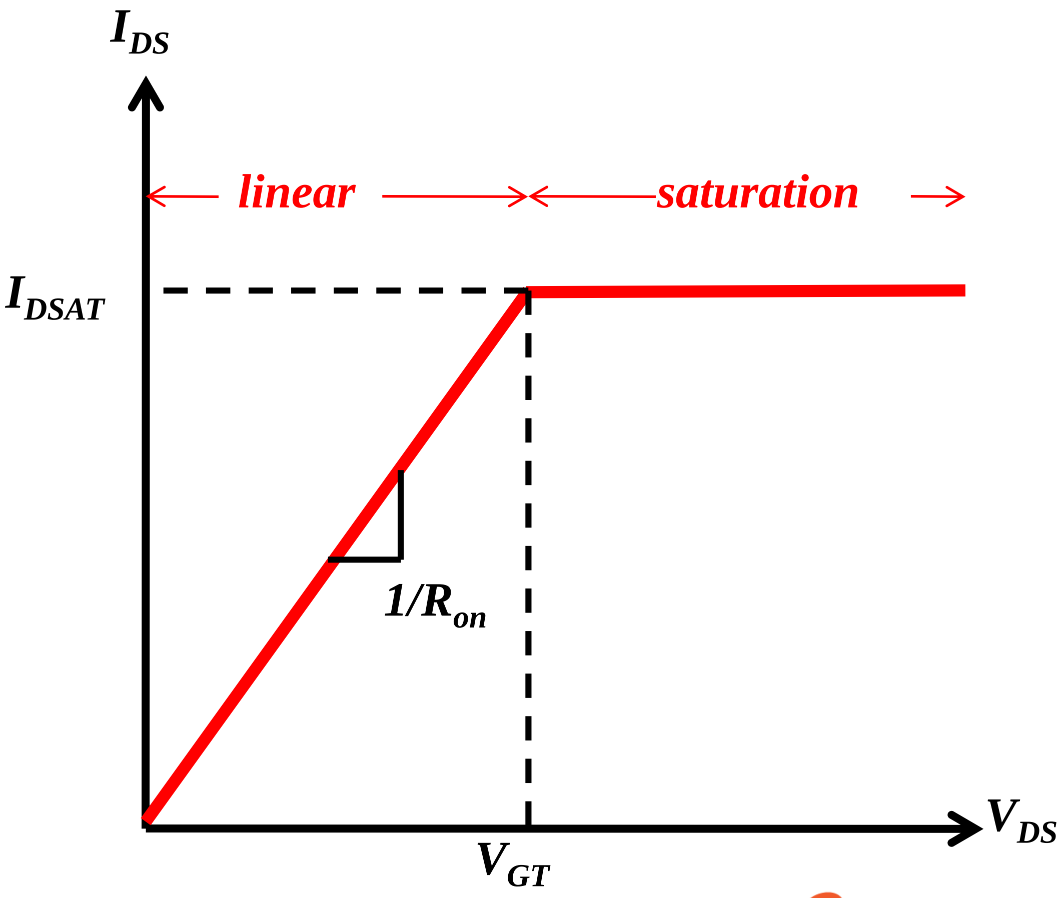


The Piece-Wise Linear Model

The Switch Model

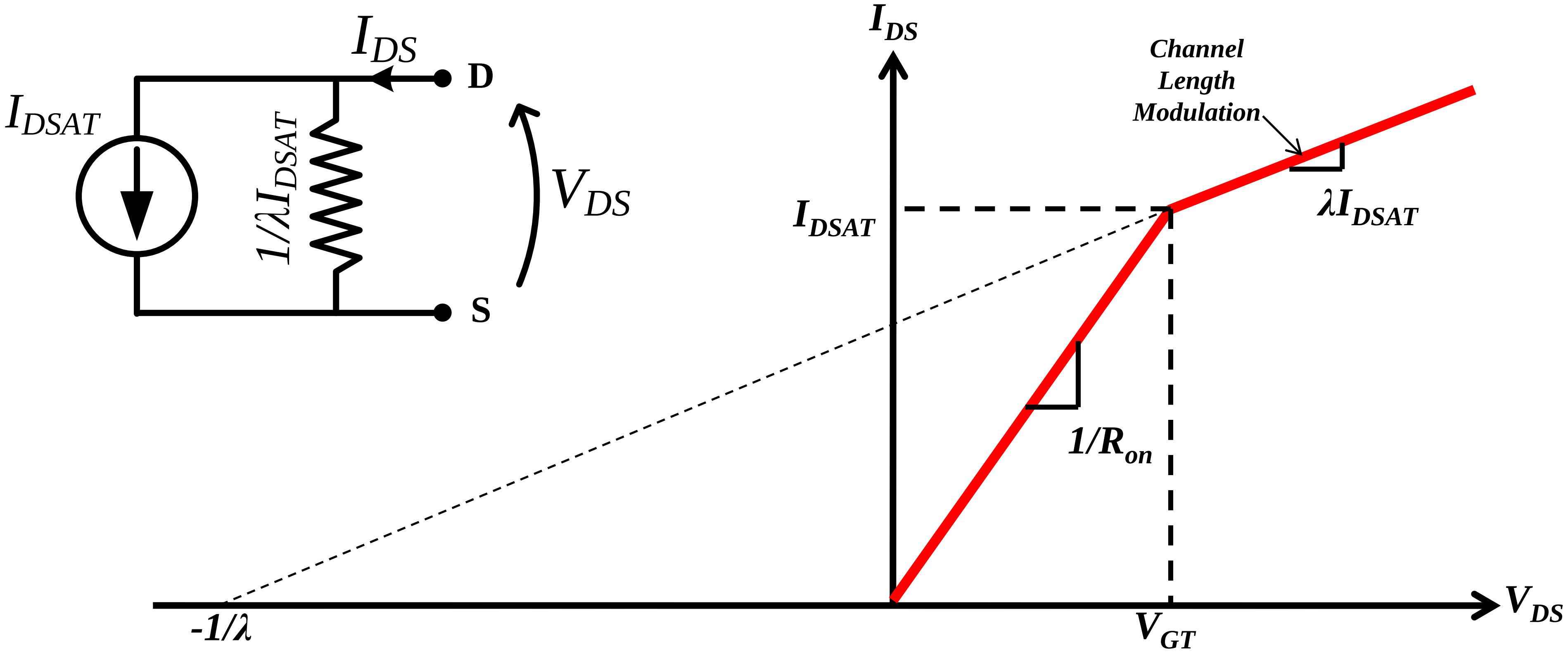
The Piece-wise Linear Model

- As we know, when the channel pinches off, the current saturates.
- This can be depicted with the simple Piece-Wise Linear Switch Model



Adding Channel Length Modulation

- **Channel Length Modulation modeled as a finite output resistance, causes a saturation current dependence on V_{DS}.**



Square Law (Shockley) Model

- **To get a more accurate model, we already are familiar with the Shockley or Square Law Model.**
- **Current is just charge times velocity, so at any point, x, along the channel:**

$$I_D(x) = -v(x)Q(x)Wdx$$

- **We found that charge can be approximated as:**

$$Q(x) = -C_{ox}\left[V_{GS} - V_{CS}(x) - V_T\right]$$

- **And the velocity is the mobility times the electrical field:**

$$v(x) = -\mu E(x) = \mu_n \frac{dV}{dx}$$

Square Law (Shockley) Model

The Switch Model

The Piece-wise Linear Model

The Square Law Model

- So we get:

$$I_D dx = \mu_n C_{ox} W \left(V_{GS} - V - V_T\right) dV$$

- And integrating from source to drain, we get

$$I_{DS} = \int_0^L I_D dx = \int_0^{V_{DS}} \mu_n C_{ox} W \left(V_{GS} - V - V_T\right) dV = \mu_n C_{ox} \frac{W}{L} V_{DS} \left(V_{GS} - V_T - \frac{1}{2} V_{DS}\right)$$

- At pinch-off ($V_{DS}=V_{GS}-V_T$), the voltage over the channel is constant, so we get:

$$I_{DSAT} = \mu_n C_{ox} \frac{W}{2L} \left(V_{GS} - V_T\right)^2$$

- This is where the "Square-Law" name comes from.

Square Law (Shockley) Model

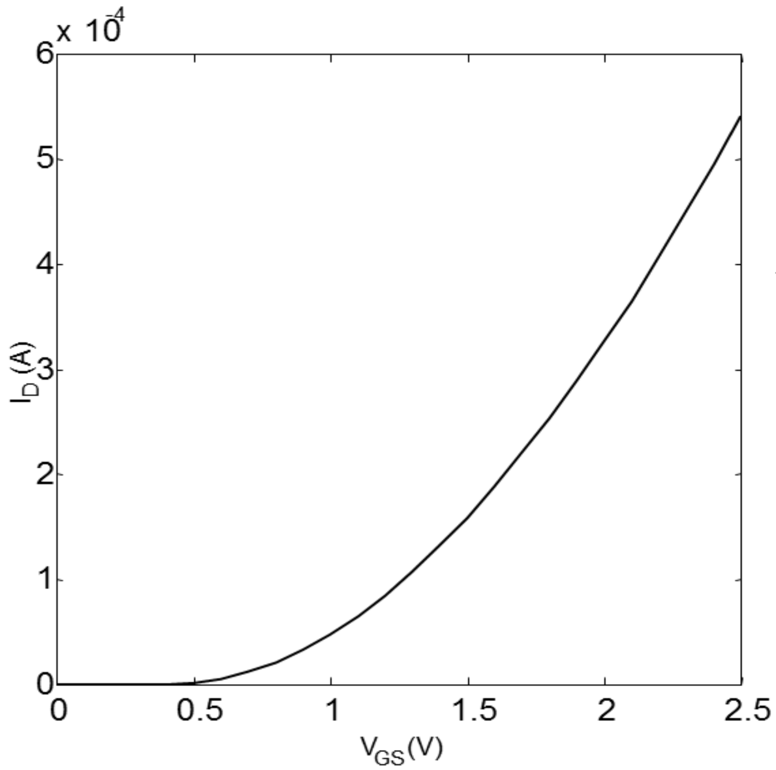
The Switch Model

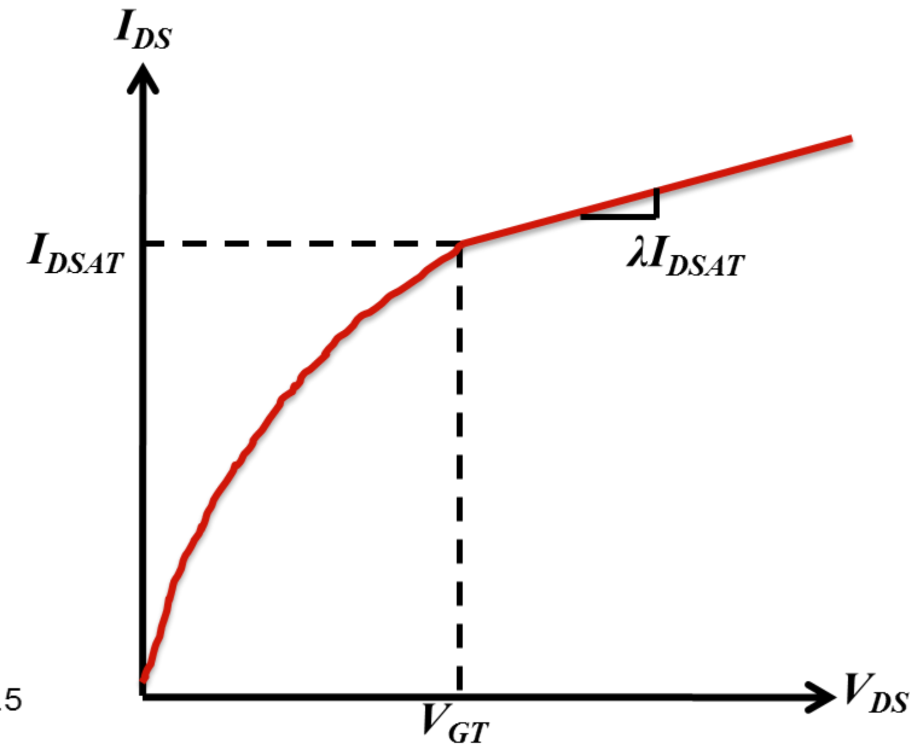
The Piece-wise Linear Model

The Square Law Model

- **Replacing V_{DS} with $V_{DSeff}=\min(V_{GS}-V_T, V_{DS})$ we get:**

$$I_{DS}=\mu_n C_{ox}\frac{W}{L}\left[(V_{GS}-V_T)V_{DSeff}-\frac{V_{DSeff}^{\,2}}{2}\right](1+\lambda V_{DS})$$

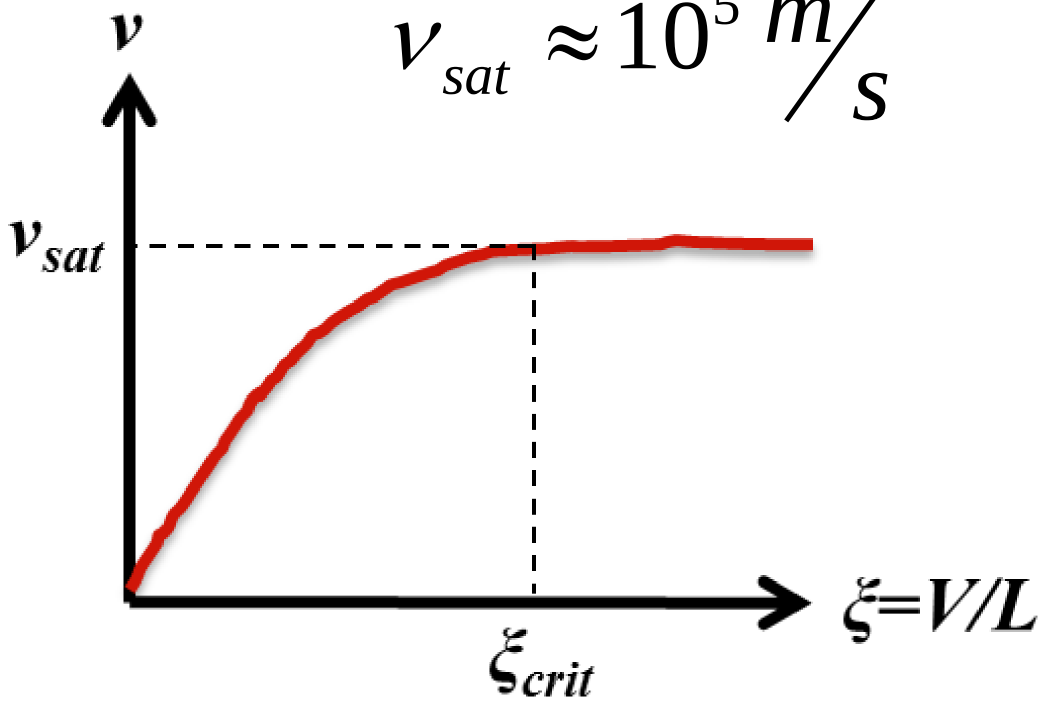


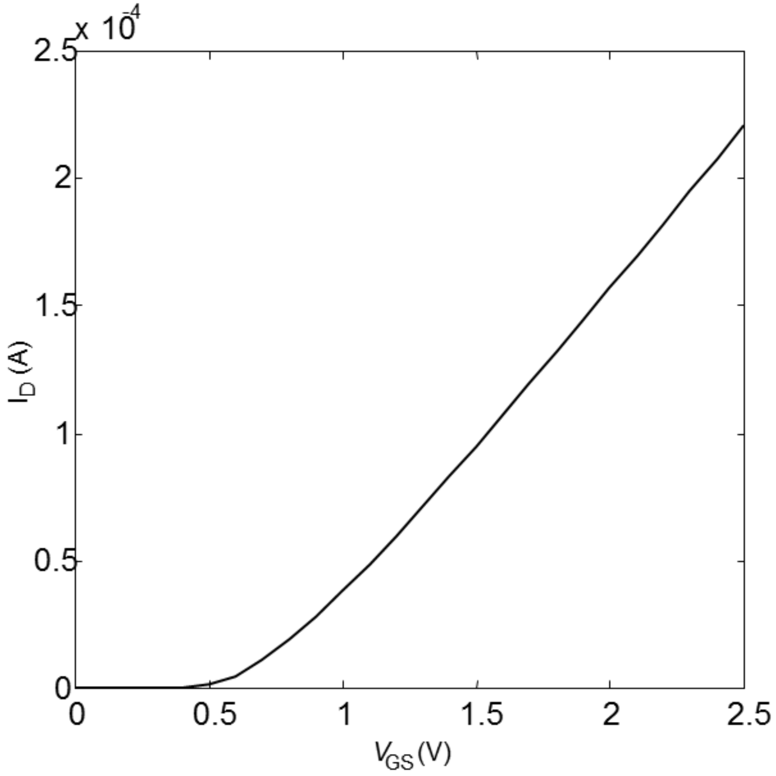


The Velocity Saturation Model

- However, when looking at a short channel device, we see a linear dependence on V_{GS}.
- This can be attributed to **Velocity Saturation**.

$$v_{sat} \approx 10^5\ {}^{m}\!/_{s}$$



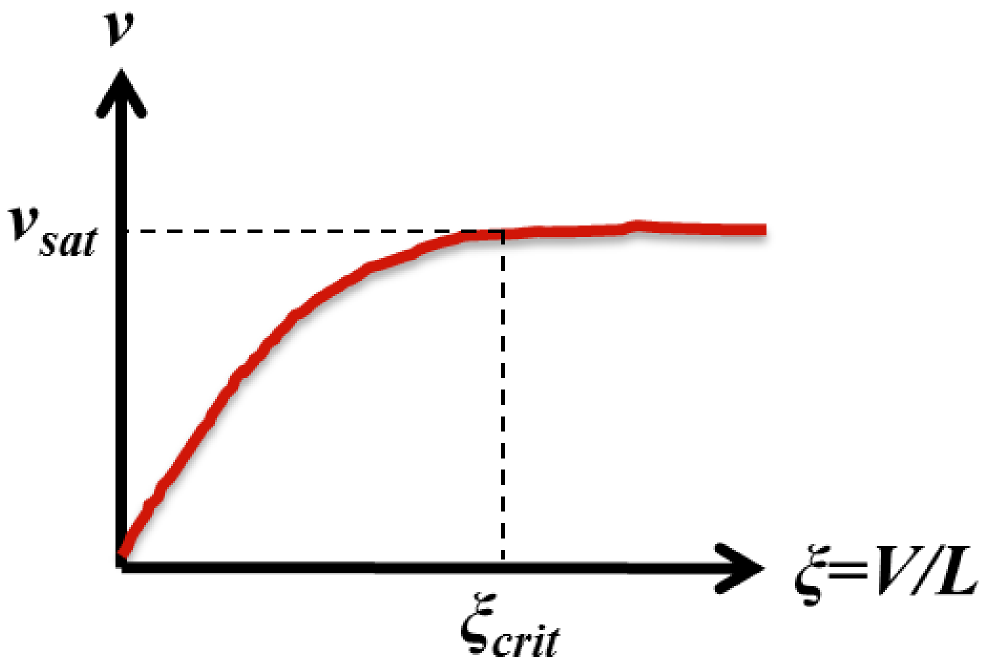


The Velocity Saturation Model

- The Switch Model
- The Piece-wise Linear Model
- The Square Law Model
- The V-Sat Model

- **A good approximation of the mobility curve is:**

$$v = \begin{cases} \dfrac{\mu\xi}{1+\xi/\xi_{crit}} & \xi < \xi_{crit} \\ v_{sat} & \xi > \xi_{crit} \end{cases}$$



- **For continuity:**

$$\xi_{crit} = \frac{2v_{sat}}{\mu}$$

- **After integration, we get:**

$$I_{DS} = \frac{\mu_n C_{ox}}{1+V_{DS}/\xi_{crit}L}\frac{W}{L}\left[\left(V_{GS}-V_T\right)V_{DS}-\frac{V_{DS}^{\ 2}}{2}\right]$$

The Velocity Saturation Model

- The Switch Model
- The Piece-wise Linear Model
- The Square Law Model
- The V-Sat Model

- **This is hard to use, but we can reach an important conclusion.**
- **We found that:**

$$I_{DS} = \frac{\mu_n C_{ox}}{1 + V_{DS} / \xi_{crit} L} \frac{W}{L} \left[\left(V_{GS} - V_T \right) V_{DS} - \frac{V_{DS}^{\ 2}}{2} \right]$$

- **And we know that for a velocity saturated device:**

$$I_{DS} = WC_{ox} \left(V_{GS} - V_{DSAT} - V_T \right) v_{sat}$$

- **Equating, we get:**

$$\boxed{V_{DSAT} = \frac{\left(V_{GS} - V_T \right) \xi_{crit} L}{\left(V_{GS} - V_T \right) + \xi_{crit} L}}$$

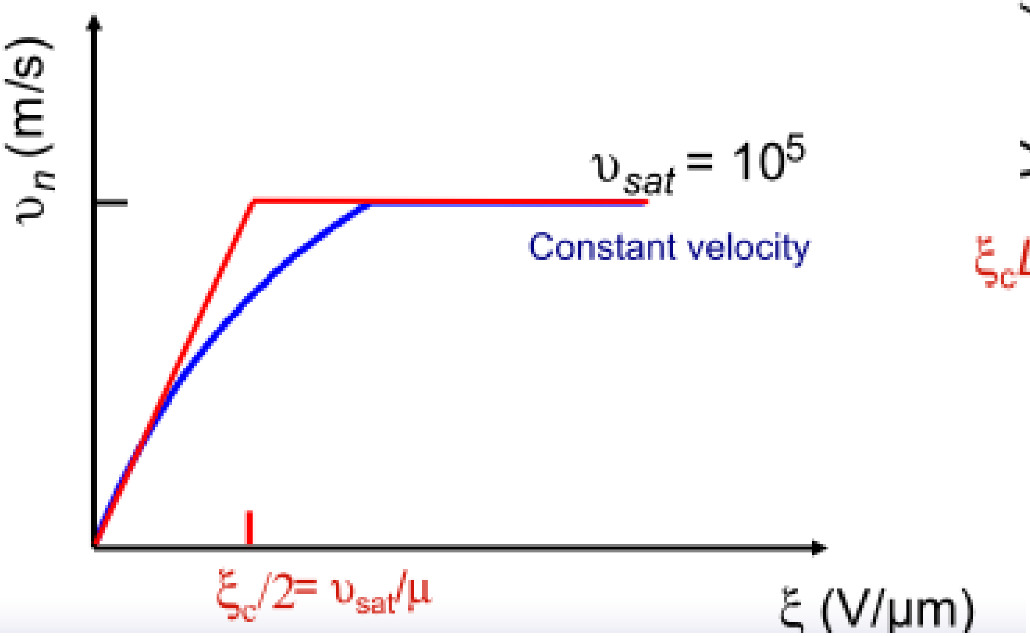
$$V_{DSAT} \left(\xi_{crit} L >> V_{GT} \right) = V_{GS} - V_T \Rightarrow pinch\,off$$

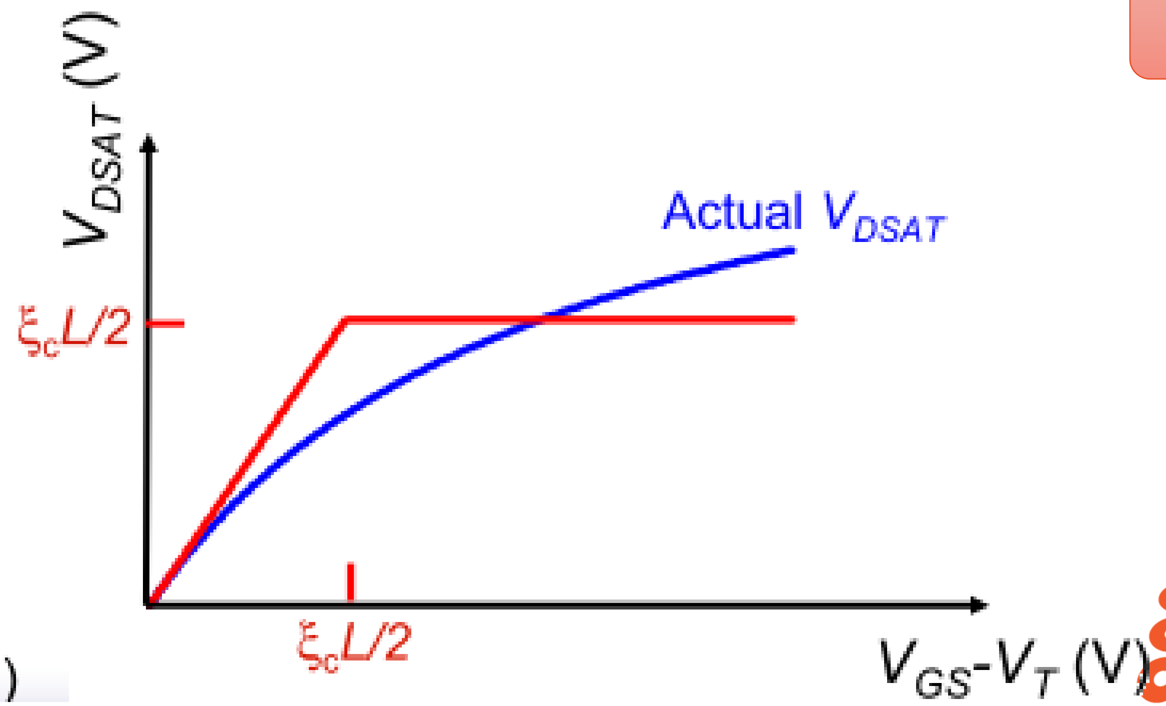
$$V_{DSAT} \left(\xi_{crit} L << V_{GT} \right) = \xi_{crit} L \Rightarrow vel\ sat$$

The Unified Model for Hand Analysis

- The Switch Model
- The Piece-wise Linear Model
- The Square Law Model
- The V-Sat Model
- The Unified Model

- **A few simple estimations will make the V-Sat model more user-friendly:**
  - The mobility is piecewise linear, saturating at $\xi > \xi_{crit}/2$
  - V_{DSAT} is piecewise linear, saturating at $V_{DSAT} = \xi_{crit} L/2$, when $V_{GT} > \xi_{crit} L/2$





The Unified Model for Hand Analysis

- **This brings us to the Unified Model:**

$$I_{DS} = \mu_n C_{ox} \frac{W}{L}\left[\left(V_{GS} - V_T\right)V_{DSeff} - \frac{V_{DSeff}^{\ 2}}{2}\right]\left(1 + \lambda V_{DS}\right)$$

$$V_{DSeff} = \min\left(V_{GS} - V_T,\ V_{DS},\ V_{DSAT}\right)$$

$$V_{DSAT} = \frac{\xi_{crit} L}{2} \qquad \xi_{crit} = \frac{2 v_{sat}}{\mu}$$

The Switch Model

The Piece-wise Linear Model

The Square Law Model

The V-Sat Model

The Unified Model

Advanced MOS Models

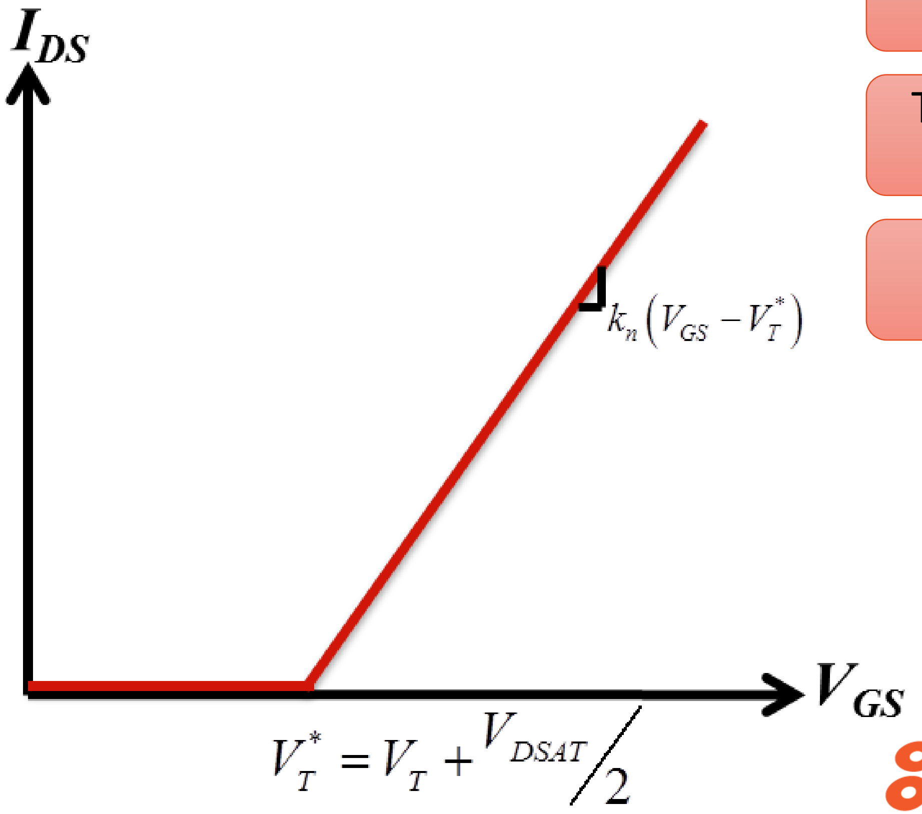
VT* Model

- **Sometimes we want to use a really simple model.**
- **We can assume that if the transistor is on, it's velocity saturated.**

$$I_{DS} = k_n\left[\left(V_{GS} - V_T\right)V_{DSAT} - \frac{V_{DSAT}^{\;2}}{2}\right] =$$

$$= k_n\left[\left(V_{GS} - \underbrace{\left(V_T + \frac{V_{DSAT}}{2}\right)}_{V_T^*}\right)V_{DSAT}\right]$$

$$\boxed{I_{DS} = \begin{cases} 0 & V_{GS} < V_T^* \\ k_n\left(V_{GS} - V_T^*\right)V_{DSAT} & V_{GS} > V_T^* \end{cases}}$$

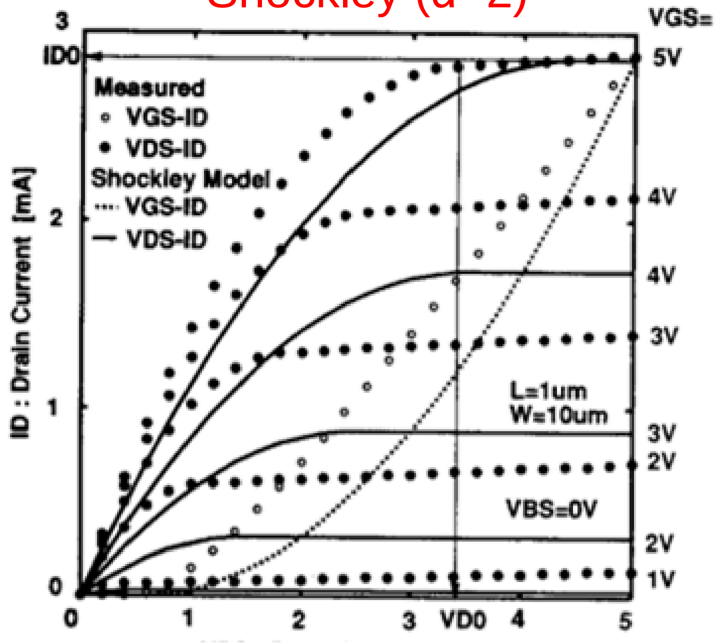


The Alpha Power Law Model

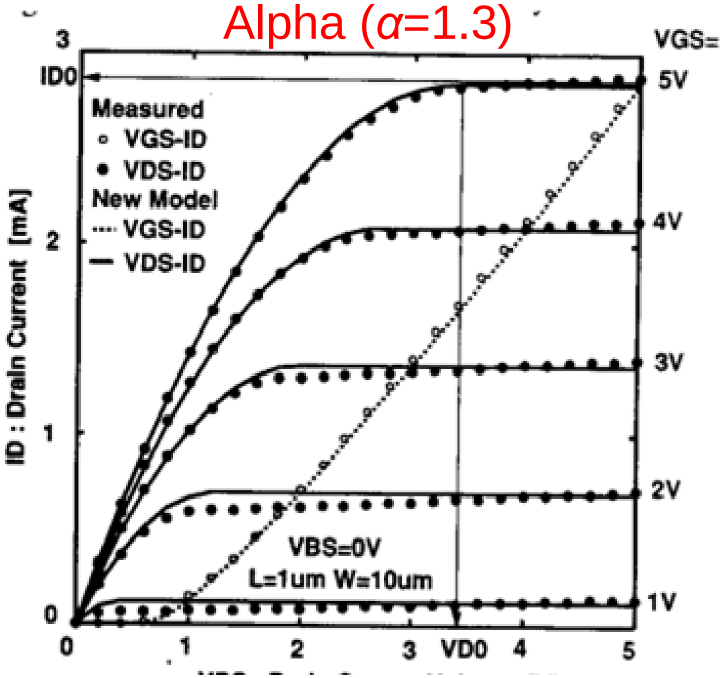
- Sakurai found that by changing the exponent of the square law, a better fit can be found with simple calculations.

$$I_{DSAT}(\alpha) = \mu_n C_{ox} \frac{W}{2L}(V_{GS} - V_T)^{\alpha}$$

Shockley (α=2)

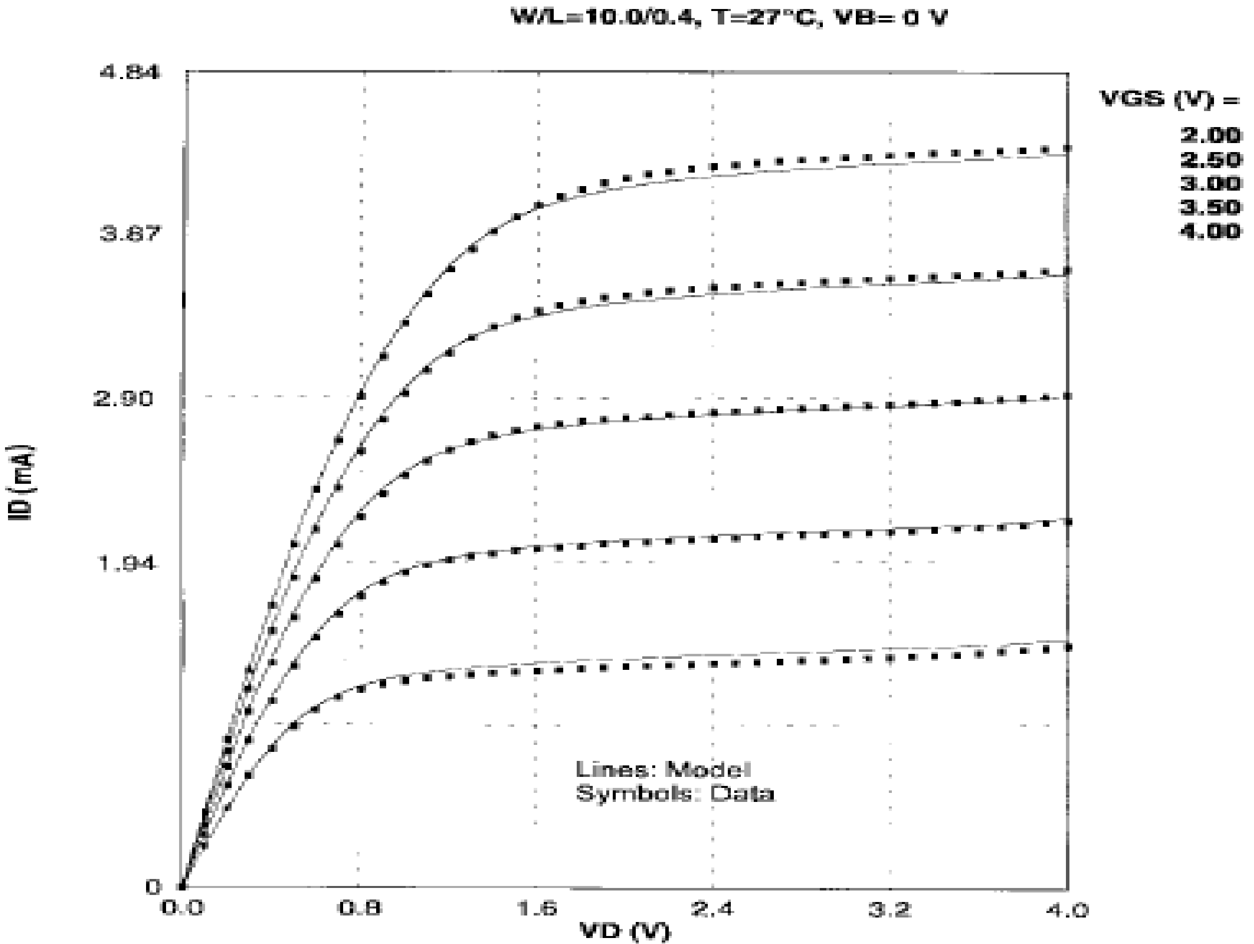


Alpha (α=1.3)



BSIM

- The main model used by simulators today is the BSIM4 model. This model uses hundreds of parameters to achieve a good fit.
- This model takes into account all known physical effects, as well as many fitting parameters.



- The Switch Model
- The Piece-wise Linear Model
- The Square Law Model
- The V-Sat Model
- The Unified Model
- The VT* Model
- The Alpha Power Law Model
- BSIM

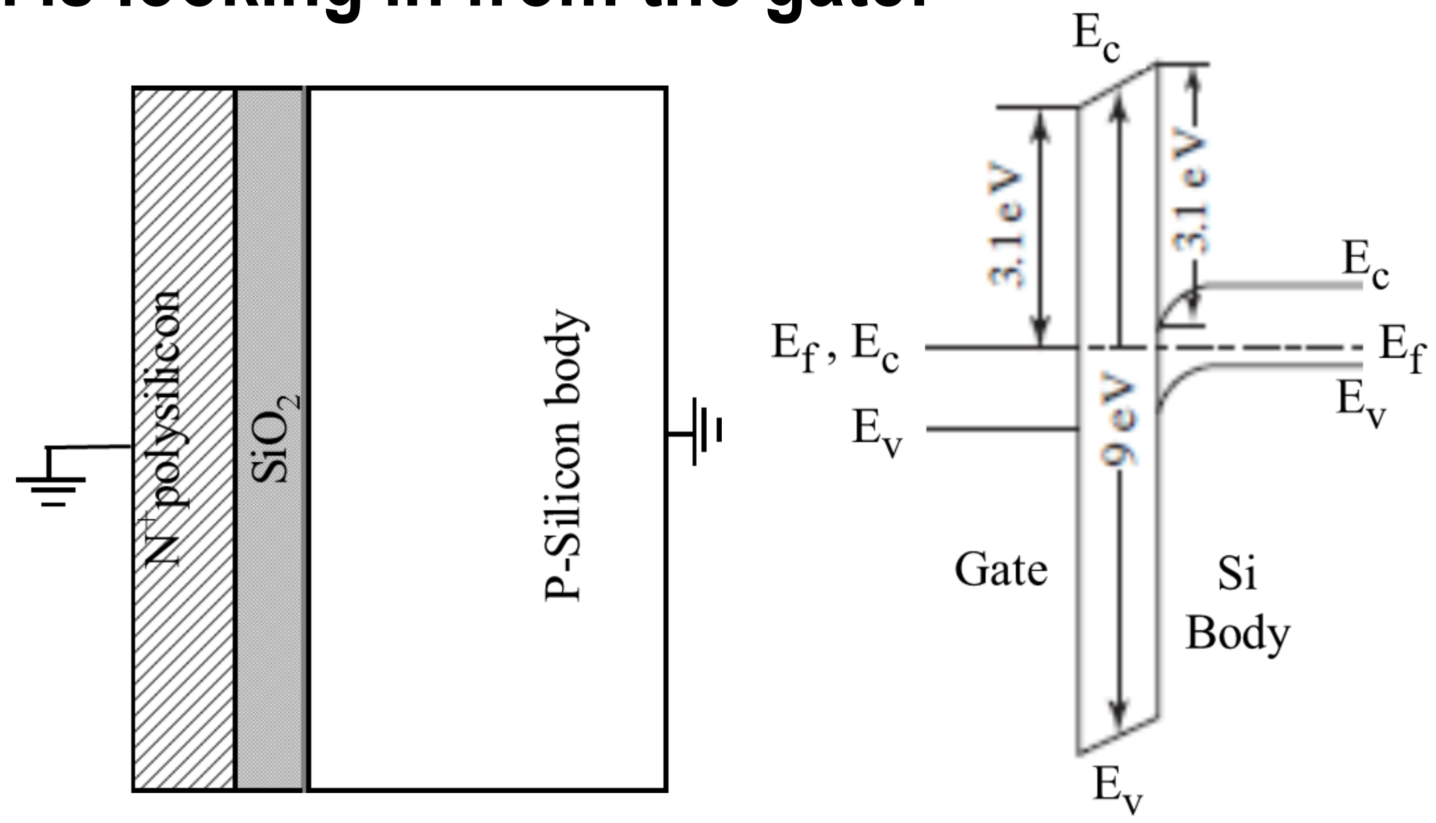
Newer Models

- **BSIM Group Webpage:**
  http://www-device.eecs.berkeley.edu/bsim/
- **Compact Model Coalition (CMC) Webpage:**
  https://www.si2.org/cmc_index.php
- **Newer BSIM Models:**
  - BSIM6 – Bulk CMOS
  - BSIMSOI - SoI model standardized in 2001 (built on BSIM3.3)
  - BSIM-CMG – multi-gate FETs (FinFet, Nanowire) – written in Verilog-A
  - BSIM-IMG – Independent Multi-Gate – for UTBB-SoI
- **EKV Model:** http://ekv.epfl.ch/
  - Developed in 1995 to provide accuracy even in subthreshold
- **PSP:** http://psp.ewi.tudelft.nl/
  - Standard for current bulk CMOS

Threshold Voltage Revisited

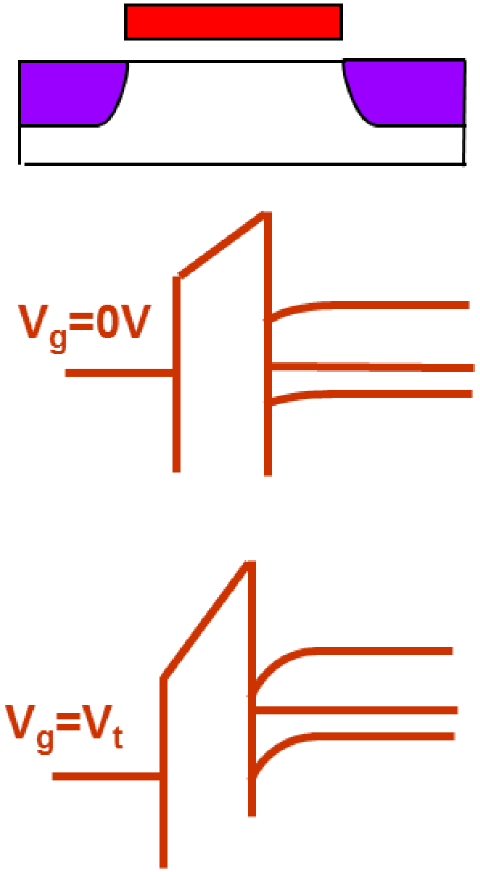
Energy Band Diagrams

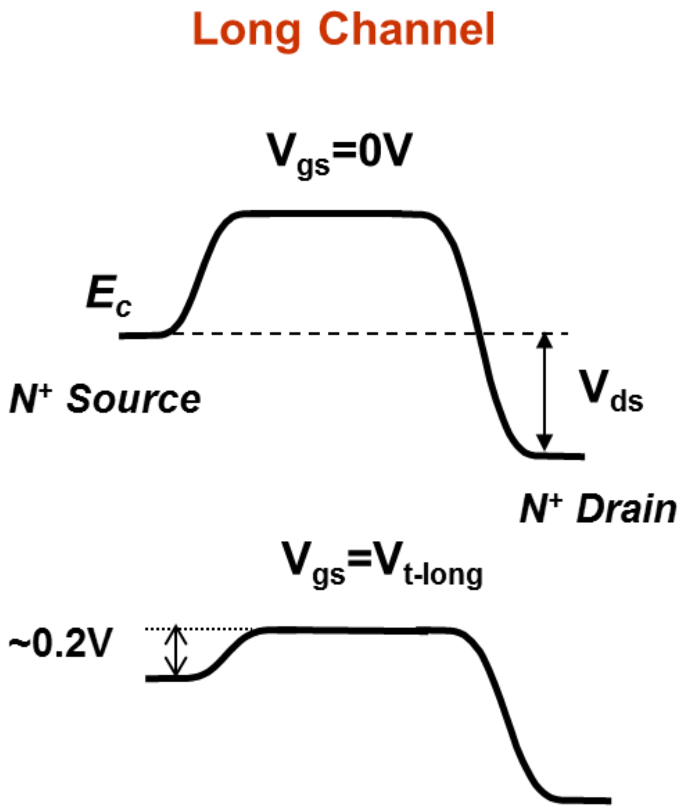
- To understand the threshold voltage and other secondary effects of the MOS device, we often use energy band diagrams.
- The first approach is looking in from the gate:



Energy Band Diagrams

- The second approach is looking from the source to the drain.





Threshold Voltage - Basic Theory

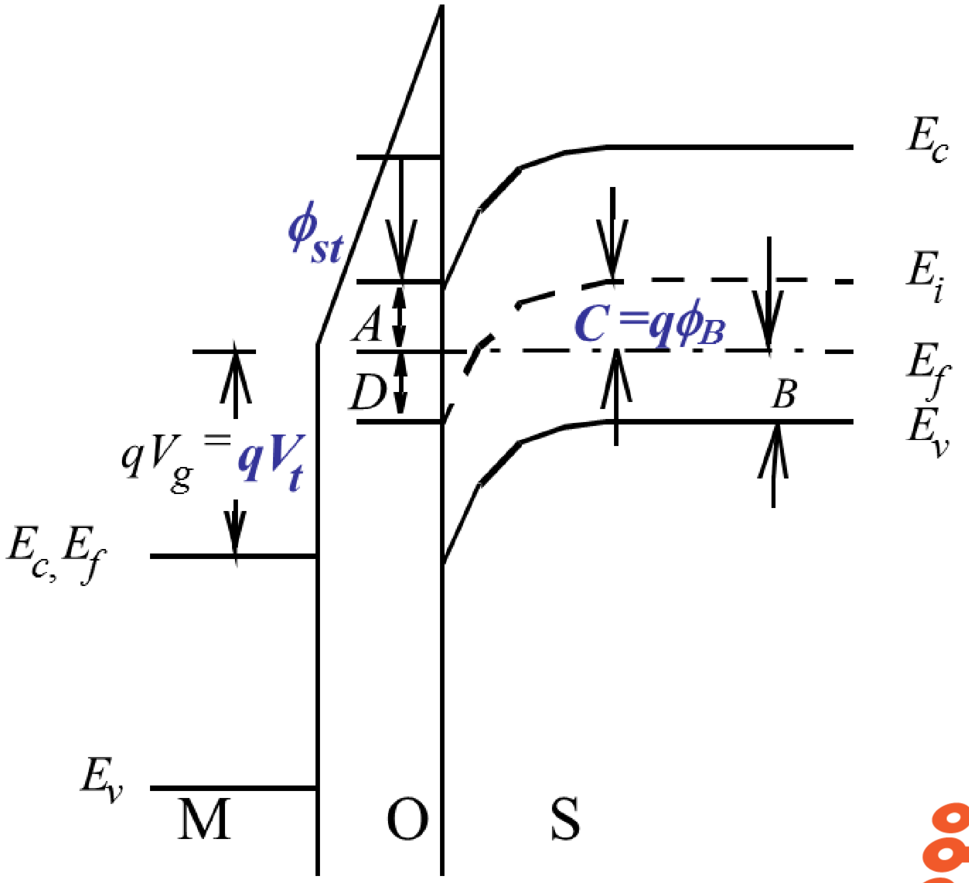
Basic Theory

- **The basic definition of threshold voltage is the gate voltage (V_G) required to invert the channel**

$$V_{T0} = \Phi_{MS} - 2\Phi_F - \frac{Q_{OX}}{C_{ox}} - \frac{Q_{dep}}{C_{ox}}$$

$$Q_{dep} = \sqrt{2qN_A\varepsilon_{si}\left|-2\Phi_F\right|}$$

$$\Phi_F = -\phi_T \ln\frac{N_A}{n_i} \qquad \phi_T \equiv \frac{kT}{q}$$



Body Effect

Basic Theory

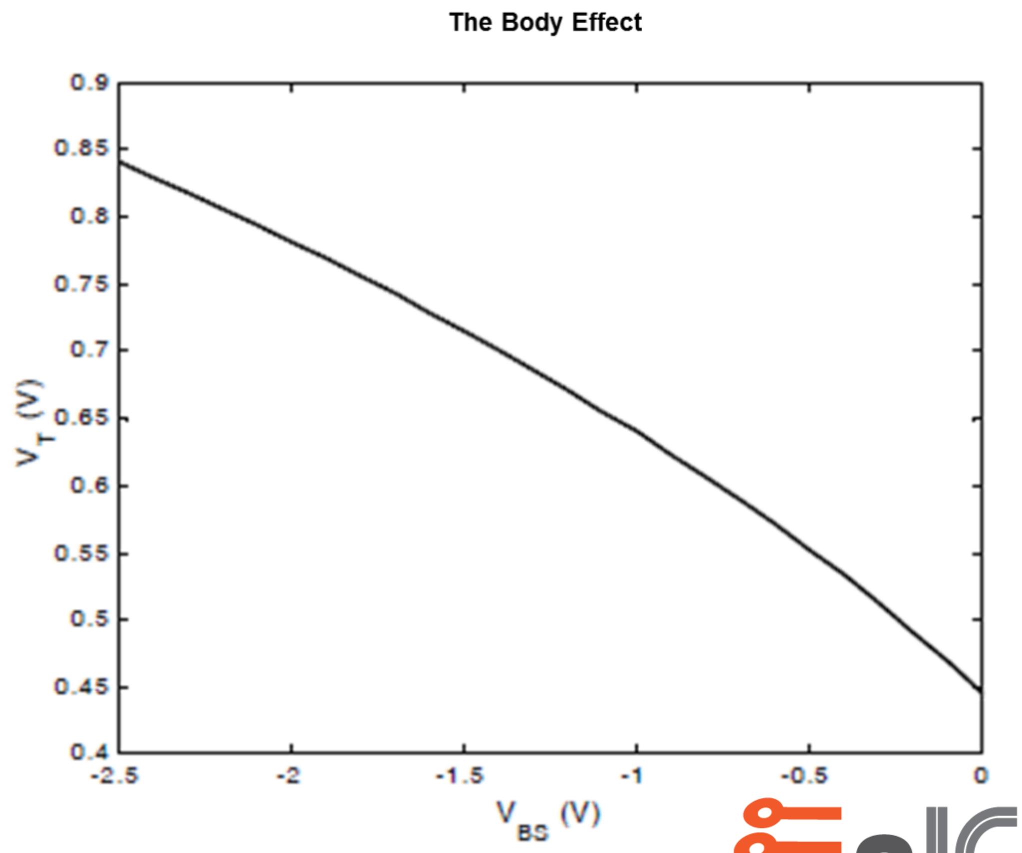
Classic Body Effect

- The appearance of a voltage difference between the **source** and **body** (V_{SB}) is known as "*The Body Effect*"
- This can be modeled by the additional charge that needs to be depleted.

$$Q_{dep} = \sqrt{2qN_A\varepsilon_{si}\left(\left|-2\Phi_F + V_{SB}\right|\right)}$$

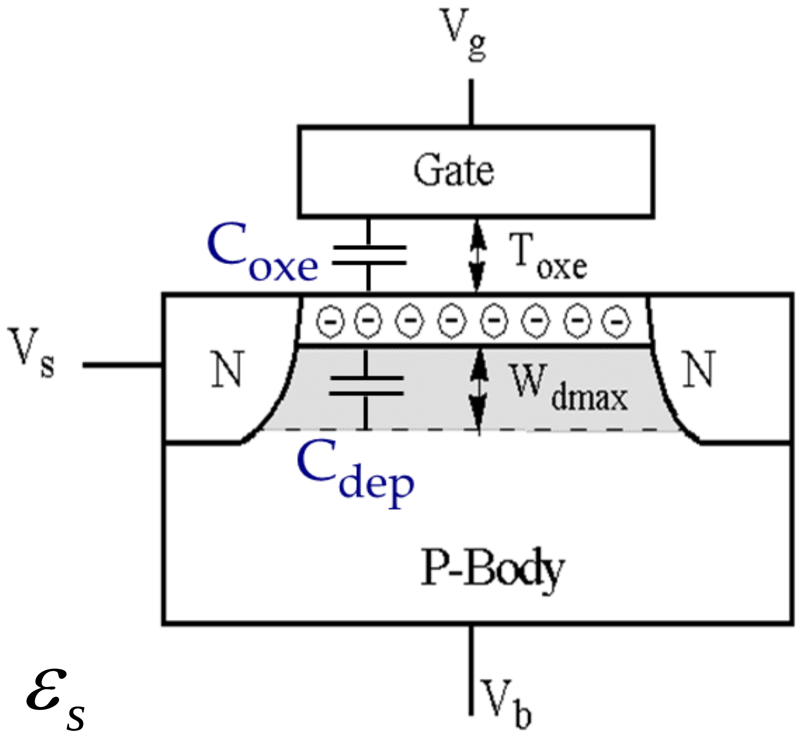
$$V_T = V_{T0} + \gamma\left(\sqrt{\left|-2\Phi_F + V_{SB}\right|} - \sqrt{\left|-2\Phi_F\right|}\right)$$

$$V_{T0} \equiv \Phi_{MS} - 2\Phi_F - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{dep0}}{C_{ox}} \qquad \gamma = \frac{\sqrt{2q\varepsilon_{si}N_A}}{C_{ox}}$$

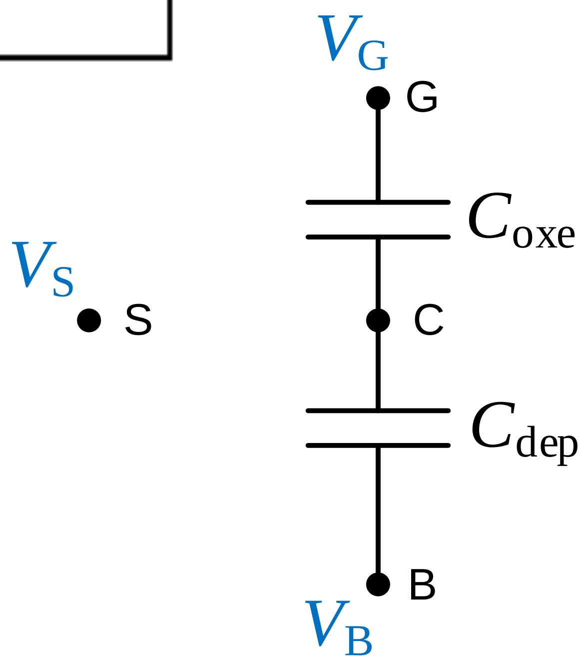


Modern Body Effect

- **A different approach is to look at the capacitive voltage divider between the gate and body (C_{GB})**



$$C_{dep} = \frac{\varepsilon_s}{W_{d\,\max}}$$



$$Q_{inv} = -C_{oxe}\left(V_{GS} - V_{CS} - V_{T0}\right) + C_{dep}\left(V_{SB} + V_{CS}\right)$$

$$= -C_{oxe}\left(V_{GS} - nV_{CS} - V_{T0}\right) \qquad n \triangleq 1 + \frac{C_{dep}}{C_{oxe}} = 1 + \frac{3T_{oxe}}{W_{d\,\max}}$$

Modern Body Effect

Basic Theory

Classic Body Effect

Modern Body Effect

- This can be shown to redefine V_T as:

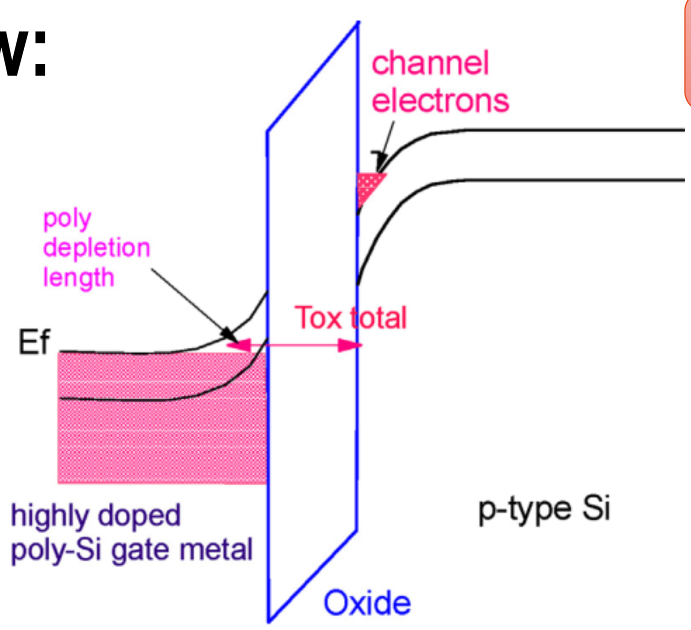
$$V_T(V_{SB}) = V_{T0} + \frac{C_{dep}}{C_{oxe}} V_{SB}$$

- In modern technologies, C_{dep}/C_{oxe} is a constant, so V_T is *linearly dependent* on V_{SB}!

Poly Depletion and Channel Depth

The threshold voltage is affected by two additional factors that we have disregarded until now:

- **Polysilicon Depletion**
  - Since polysilicon is, itself, a semiconductor, the depletion layer into the poly effectively increases the oxide thickness.
- **Channel Depth**
  - Since the channel is not a 2-dimensional line along the surface, the oxide thickness is essentially increased.



$$n \triangleq 1+\frac{C_{dep}}{C_{oxe}}=1+\frac{3T_{oxe}}{W_{d\max}}$$

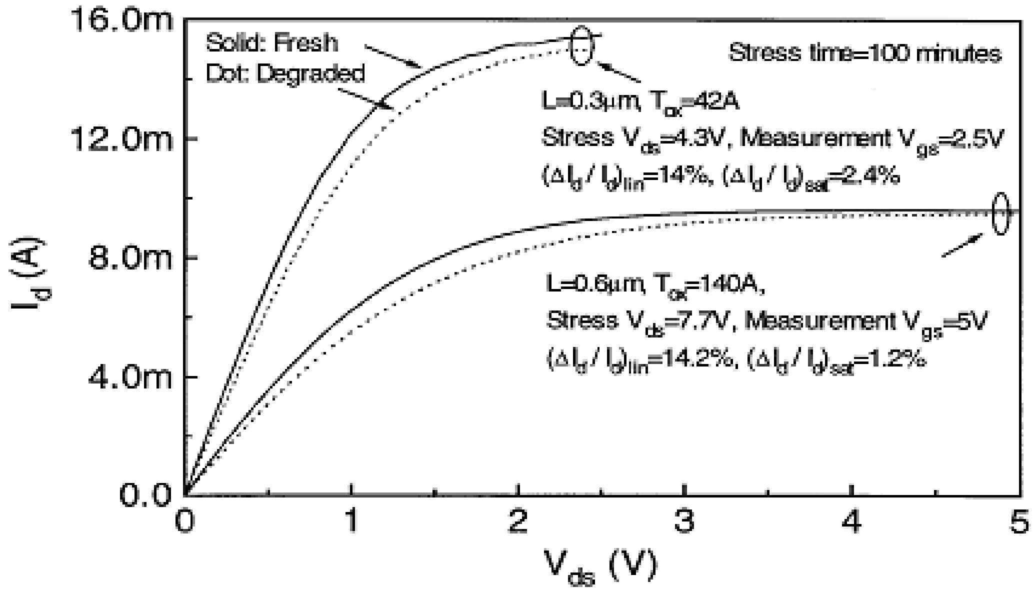
Hot Carrier Effects

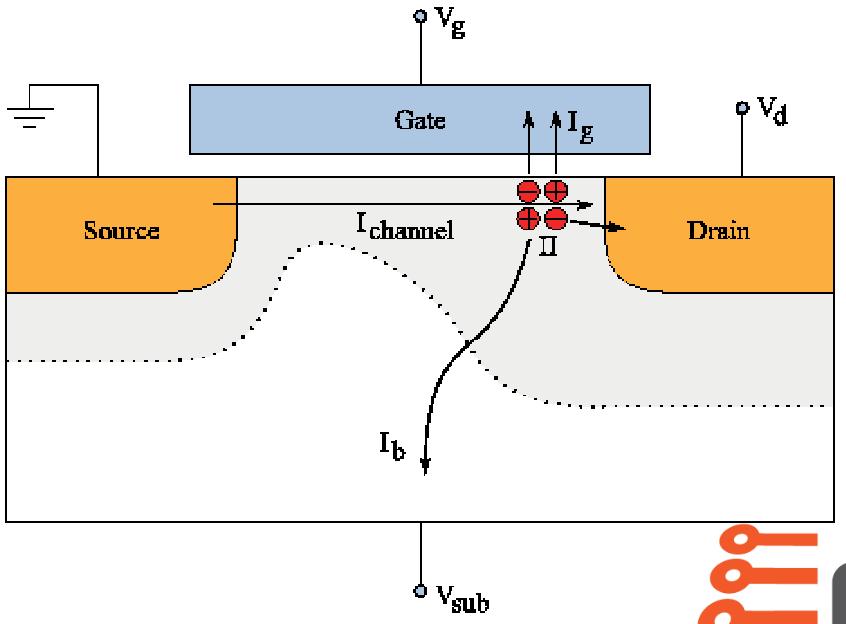
- Basic Theory
- Classic Body Effect
- Modern Body Effect
- Hot Carriers

- **Electrons can get so fast that they can tunnel into the gate oxide and increase the threshold voltage.**

$$V_{T0} = \Phi_{MS} - 2\Phi_F - \frac{Q_{OX}}{C_{ox}} - \frac{Q_{dep}}{C_{ox}}$$

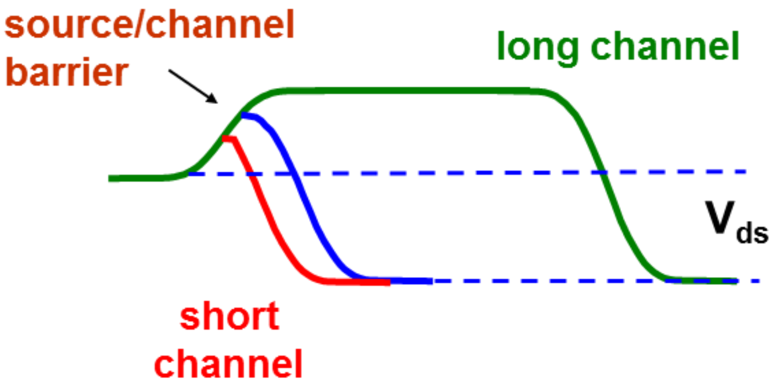
- **This is a reliability issue as it happens over time.**

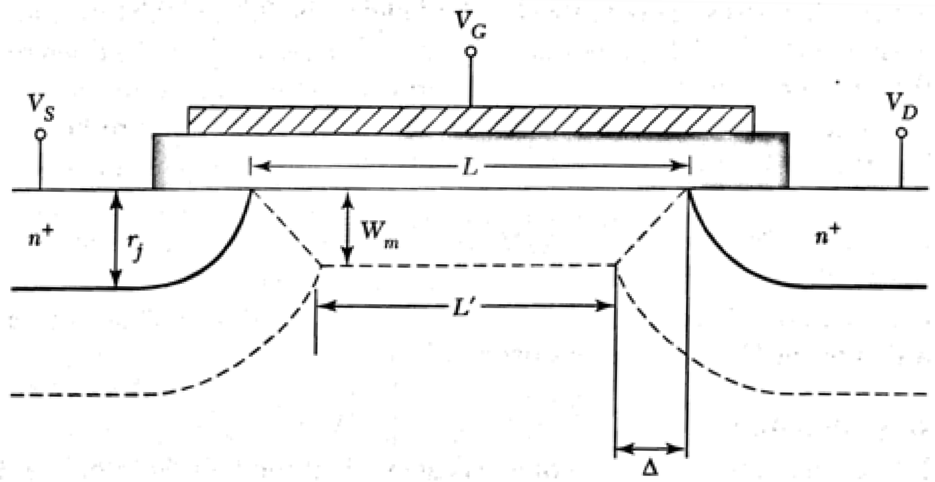




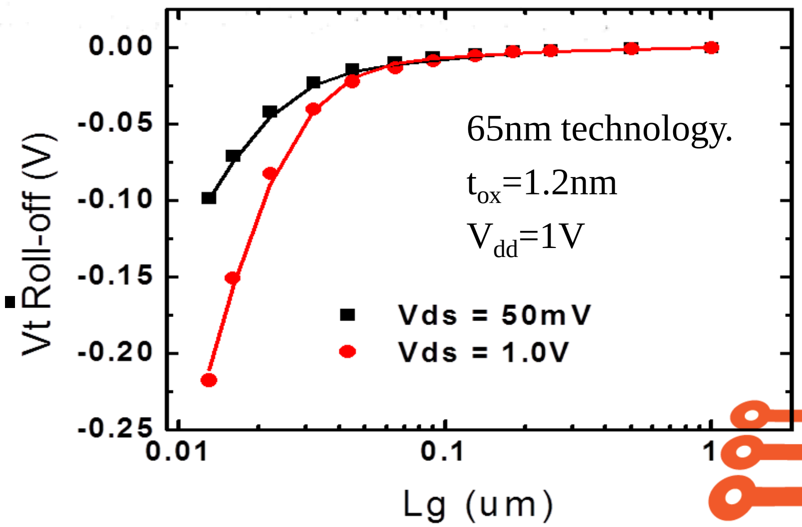
V_T Roll Off (Short Channel Effect)

- As channel length is reduced, effective channel length is reduced by depletion regions.





- A trapezoid is created under the gate, dividing the channel into the region controlled by the gates and by the drain.
- In essence, V_T is reduced.

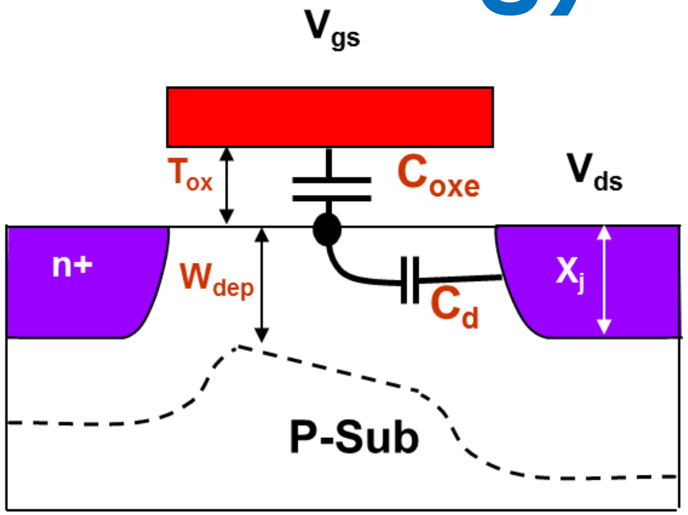


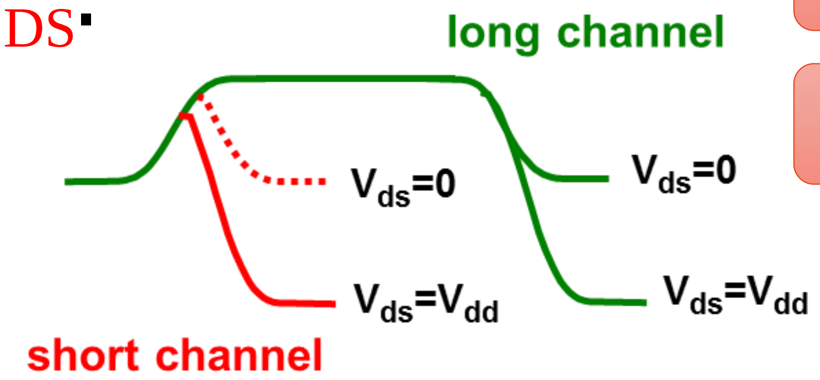
K. Goto et al., IEDM 2003

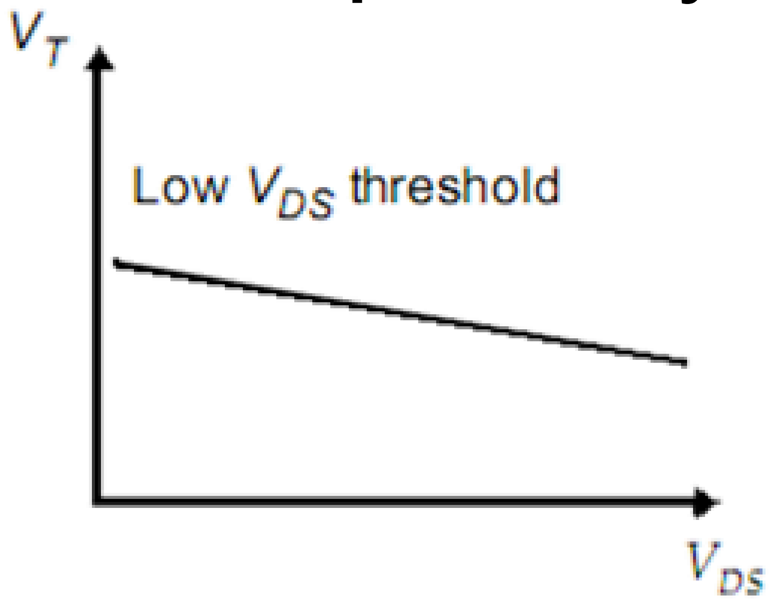
- Basic Theory
- Classic Body Effect
- Modern Body Effect
- Hot Carriers
- VT Roll-Off

DIBL (Drain Induced Barrier Lowering)

- **In short channels, the barrier of the channel is essentially lowered, as the drain causes the energy band to drop closer to the source.**
- **This is exponentially dependent on V_{DS}.**

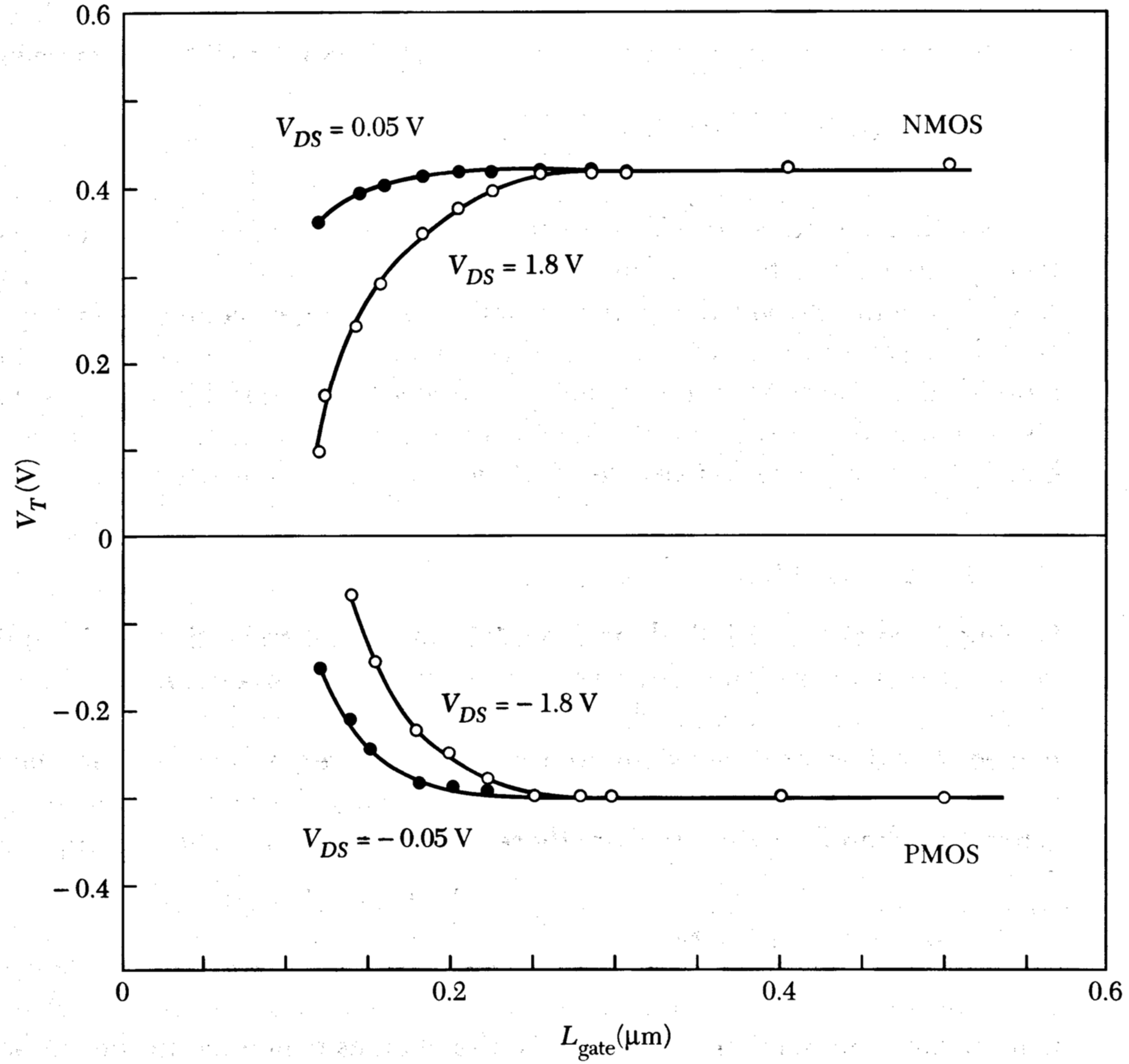






$$V_T = V_{T,long} - (V_{DS} + 0.4) \cdot \frac{C_d}{C_{oxe}}$$

Roll Off / DIBL combined



Basic Theory

Classic Body Effect

Modern Body Effect

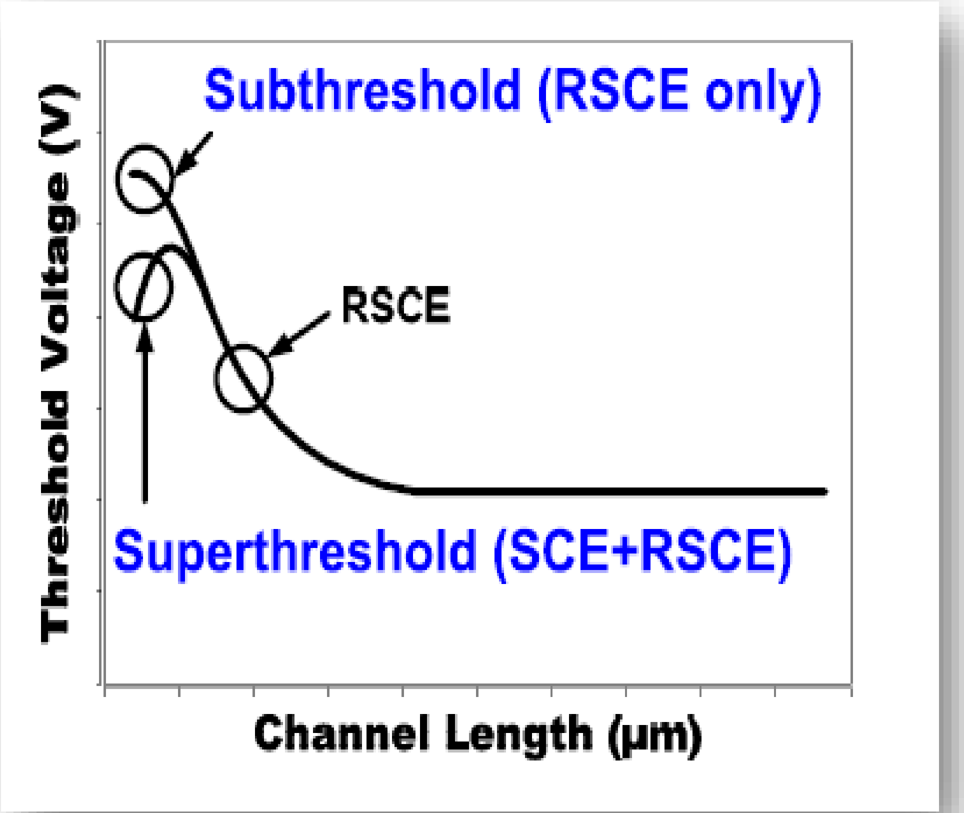
Hot Carriers

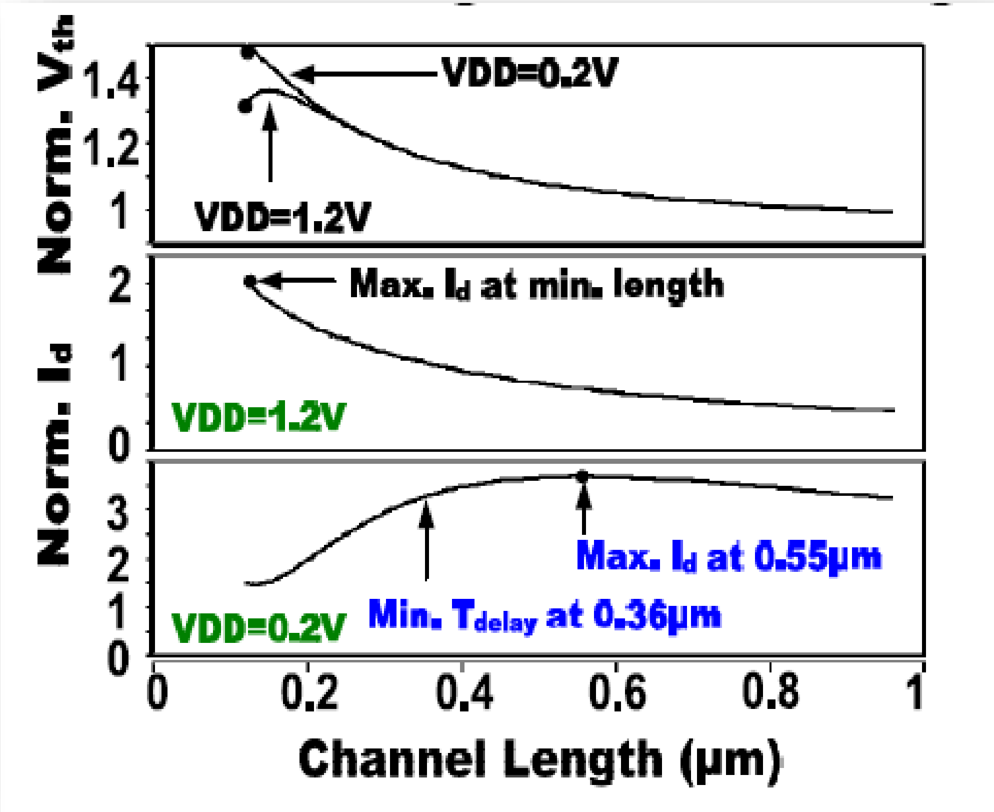
VT Roll-Off

DIBL

Reverse Short Channel Effect (RSCE)

- V_T actually ***increases*** at channel lengths a bit higher than minimum…





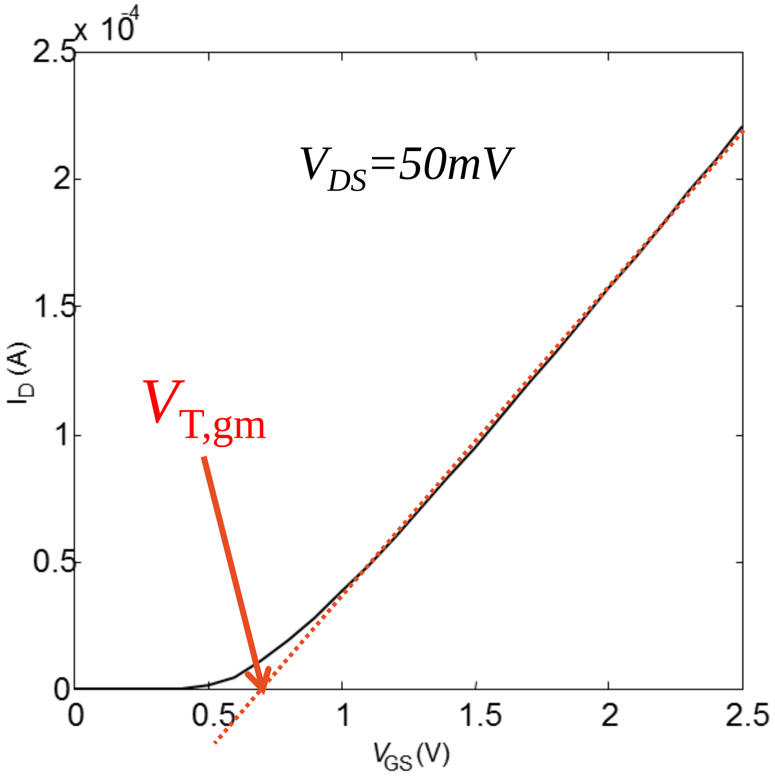
- Basic Theory
- Classic Body Effect
- Modern Body Effect
- Hot Carriers
- VT Roll-Off
- DIBL
- RSCE

How to Measure VT

- There are various ways to measure V_{T}
- One classic way takes a small V_{DS} and sweeps V_{GS}.

$$I_d = k\left[(V_{gs} - V_t)V_{ds} - \cancel{0.5V_{ds}^2}\right] \propto V_{gs} - V_t$$

- So we can find the V_{GS} at which the linear part crosses $I_{\mathrm{ds}}=0$.



Basic Theory

Classic Body Effect

Modern Body Effect

Hot Carriers

VT Roll-Off

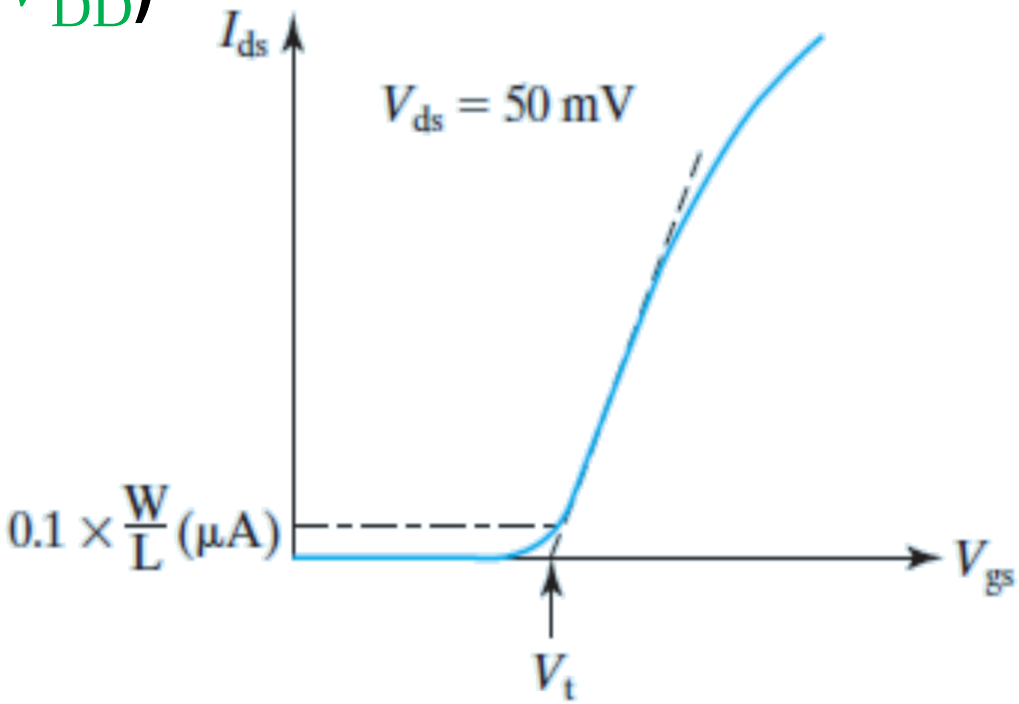
DIBL

RSCE

Measuring VT

How to Measure VT

- **One of the more common ways is to find the V_{GS} at which I_{DS}=100 nA x W/L.**
- **For $V_{T,lin}$, set a low V_{DS} (V_{DS}=50mV)**
- **For $V_{T,sat}$ set a high V_{DS} (V_{DS}=V_{DD})**



EnICS

OP and MP in Spectre

Further Reading

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- Weste, Harris “CMOS VLSI Design”, Chapter 7
- C. Hu, “Modern Semiconductor Devices for Integrated Circuits”, 2010, Chapters 4-7
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- A. B. Bhattacharyya “Compact MOSFET models for VLSI design”, 2009,
- T. Sakurai, “Alpha Power-Law MOS Model” – JSSC Newsletter Oct 2004
- Managing Process Variation in Intel’s 45nm CMOS Technology, Intel Technology Journal, 2008
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