

Digital Microelectronic Circuits (361-1-3021)

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Lecture 6: CMOS Digital Logic



Digital Microelectronic Circuits

Last Lectures

- □ The CMOS Inverter
- □ CMOS Capacitance
- Driving a Load



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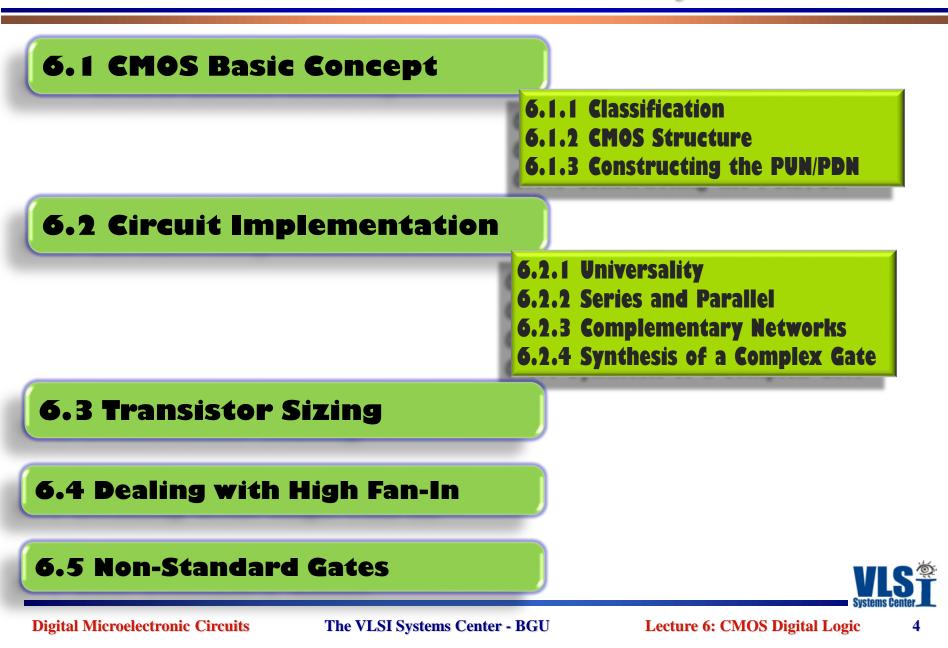
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This Lecture

- Now that we know all about an inverter, you are probably asking "So what can we do with it"?
- Well, with an inverter by itself, you can't do much, but the principles in constructing and analyzing the CMOS inverter are the same as those used to make up the whole CMOS digital logic family.
- During this lecture, we will learn how to compile any combinational Boolean function from CMOS logic and analyze the pros and cons.



What will we learn today?





6.1 CMOS Basic Concept

6.2 Circuit Implementation

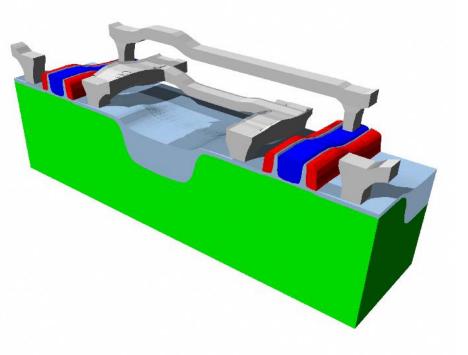
6.3 Transistor Sizing

6.4 Dealing with High Fan-In

6.5 Non-Standard Gates



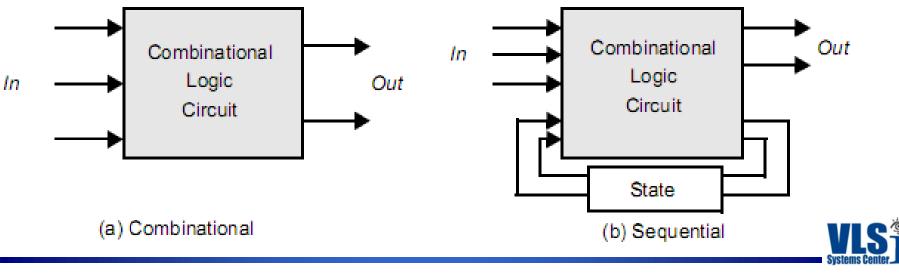
CMOS BASIC CONCEPT





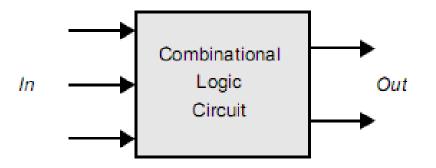
Classification

- A first classification of logic circuits or logic gates is *Combinational* vs. *Sequential* operation.
 - » Combinational logic (a.k.a. non-regenerative) circuits are characterized by an output that at any point in time is a function of the current inputs by some Boolean expression.
 - » *Sequential* (a.k.a. *regenerative*) circuits, are a function of the current inputs as well as previous states through feedback.



Classification

□ The focus of this lecture is *Static CMOS* or *Complementary CMOS* implementation of *Combinational Logic*.



□ Sequential circuits will be discussed in a later lecture.



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Another classification of logic circuits is *Static* vs.
Dynamic Logic Circuits.

- » Static Circuits assume the value of the Boolean function implemented by the circuit at all times (after the initial transient).
- » *Dynamic Circuits temporarily* store a predefined value, and only show the function output *after a fixed interval*.



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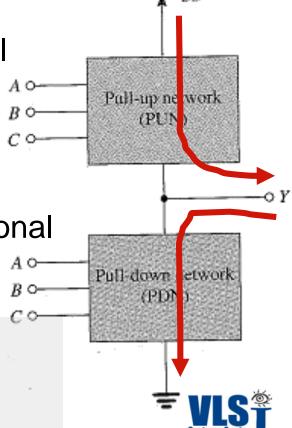
- Static Circuits are easier to implement and provide lower power consumption when the activity factor is low.
- Dynamic Circuits provide faster and smaller gates, but design and operation are more complex and they are mores sensitive to noise.
- This lecture will discuss *Static* or *Complementary CMOS*, obviously belonging to the *Static* classification.



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□ Let's remember the concepts of a *Pull Up Network* (*PUN*) and *Pull Down Network* (*PDN*) that we discussed in *Lecture 2*: V_{DD}

- » A *Pull Up Network* is a combinational block that presents a high output level ^A when activated.
- » A Pull Down Network is a combinational block that presents a low output level when activated.

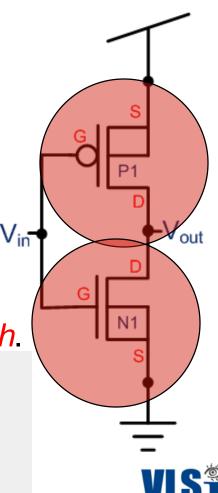


CMOS Structure

□ In the case of the *CMOS inverter*:

» The **PUN** was the **pMOS** that connected V_{DD} to the output when the input was **low**.

The PDN was the nMOS that connected GND to the output when the input was high.



□ Looking more closely at the *CMOS inverter*, we can see that:

- » Applying a *positive value* to the input opened the *PDN* and closed the *PUN*.
- » Applying a *negative value* to the input closed the *PDN* and opened the *PUN*.

Essentially, applying the same input to the PUN and PDN resulted in an opposite reaction

This is the basic characteristic of CMOS – complementary pull up and pull down networks.



out

□ Extending the concept beyond the inverter:

- » We can apply any *N-inputs* to both the *PUN* and *PDN*.
- » The logic functions of the PUN and PDN will be complementary so all input combinations will result in one of the two open and the other closed.
- » If the output is meant to be '1', the PUN will be open and the PDN will be closed, resulting in the propagation of V_{DD} to the output.
- » If the output is meant to be '0' the PDN will be open and the PUN will be closed, resulting in the propagation of GND to the output.

Pall-

Pull

RÓ

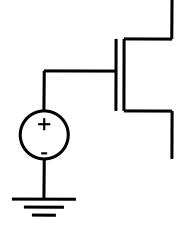
 $A \circ$

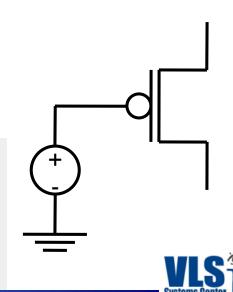
 $B \circ$

 $C \circ$

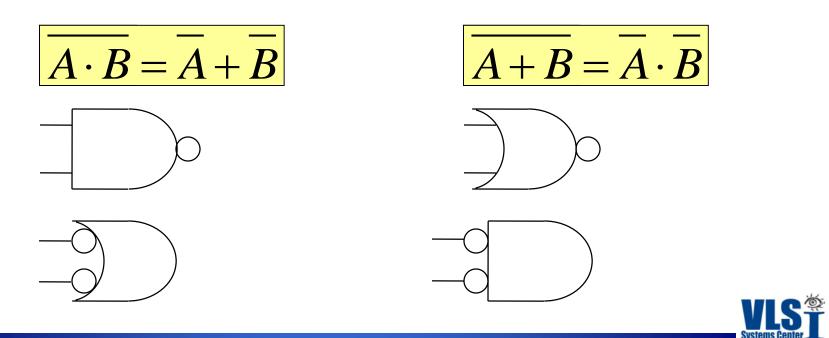
How do we construct the *PUN* and *PDN*?
Let's look at a *MOSFET* transistor as a *voltage-controlled switch*:

- » An *nMOS* is *on* when the input is *high* and *off* when it is *low*.
- » A pMOS is an inverted nMOS on when the input is low and off when the input is high.





- Taking this into account, we need to create complementary reactions to the same inputs.
 - » For example, let's look at the *NAND* and *NOR* functions.
 - » Using inverted inputs, we can arrive at the same logic function:



Interestingly, we have a perfect solution to produce these complementary logic expressions.

□ Remember our good old friend *DeMorgan*.

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$
$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

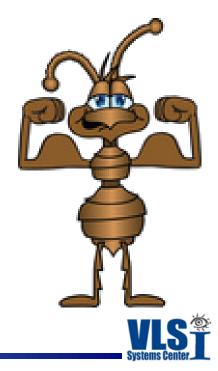


Don't *DeMorgan's Equations* look just like the expressions on the last slide?

Using *DeMorgan*, we are able to make a *Pull Up* and *Pull Down Network* of the same *Boolean Function*!



- Now comes the question of who to put on *top* and who on the *bottom*...
 - » Using our engineering intuition, we can look at the CMOS inverter and realize that the pMOS is probably in the Pull Up...
 - » But Why???
- The answer lies in a very important characteristic of the MOSFET transistor.
 - » A *pMOS* is a device that generates "Strong Ones".
 - » An *nMOS* is a device that generates "Strong Zeros".

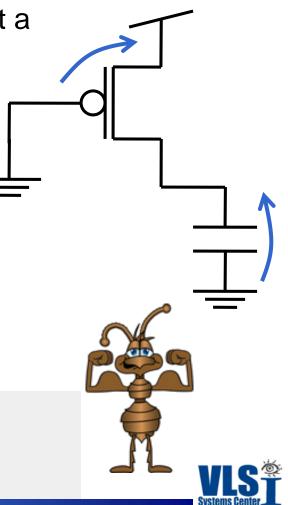


Constructing the PUN/PDN

□ So what are "*Strong Ones*" and "*Strong Zeros*"?

- » Let's see what happens when we connect a pMOS in a Pull Up Network:
- » As we can see, $V_{SG}=V_{DD}>/V_{Tp}/$ independent of the output voltage.
- » Therefore, the output capacitance will load until $V_{out}=V_{DD}$ causing $V_{SD}=0$.

□ We've provided a "*Strong One*".

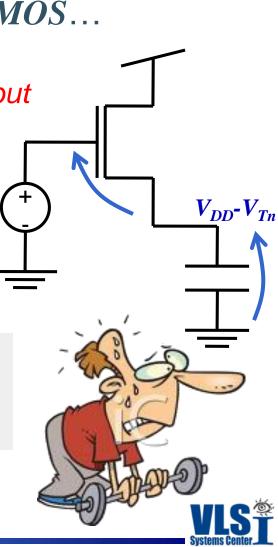


Constructing the PUN/PDN

\Box On the other hand, if we connect an *nMOS*...

- » Now, $V_{GS}=V_{DD}-V_{out}$, in other words, the state of the transistor *depends on the output voltage*.
- » Once the output reaches V_{DD} - V_{Tn} , the transistor *switches off*, and the output capacitance *stops charging*.

□ We've provided a "*Weak One*".

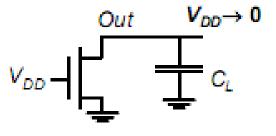


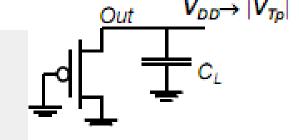
Obviously, we'll prefer pMOS transistors in our Pull Up Networks!

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□ The same can be shown for a Pull Down Network.

- » Here, an *nMOS* provides a "*Strong Zero*", as $V_{GS}=V_{DD}$ independent of the output voltage.
- » A *pMOS*, on the other hand, provides a "*Weak Zero*", only charging the output to V_{DD} -/ V_{Tp} /.





We will use nMOS transistors in our Pull Down Networks!



□ To summarize, we saw that:

- » To get a "*Strong One*", we need *pMOS* transistors in the *PUN*.
- » To get a "*Strong Zero*", we need *nMOS* transistors in the *PDN*.
- This is the basic concept in constructing a *Static CMOS* logic gate.
- □ Again, we can look at the *CMOS Inverter* and see how our basic concept is used.

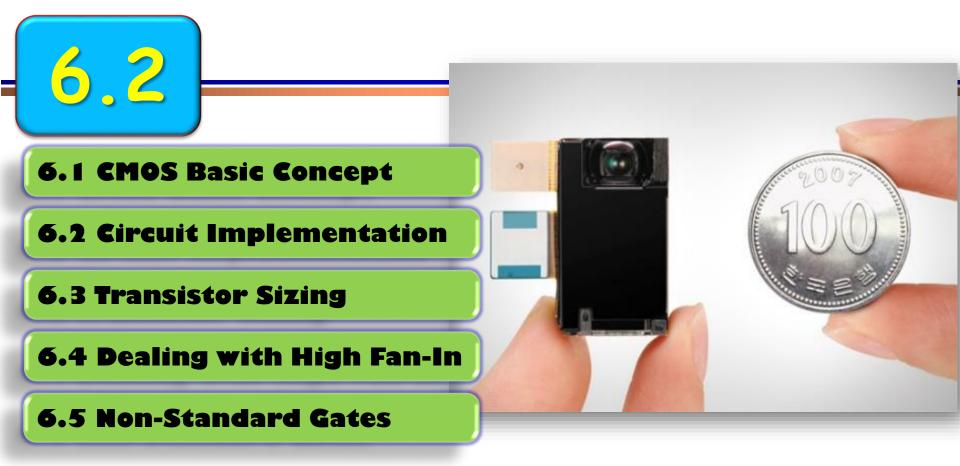
Summary

□ Now we know:

- » How to write our logic functions (DeMorgan)
- » What our building blocks are (PUN=pMOS, PDN=nMOS).

The only thing we're missing is how to implement the functions in the PUN and PDN.





Okay, we understand the concept. It's time for

CMOS DIGITAL LOGIC IMPLEMENTATION



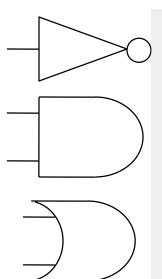
Universality

According to Boolean Algebra, to make any function, we need a universal set, comprising:

» An Inverter (NOT)

» An AND function

» An **OR** function

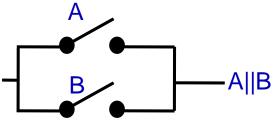


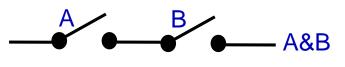
Luckily, they are very easy to implement using MOSFETs.



Back in Lecture 3, we discussed the "Switching Concept":

- » Connecting Switches in *Parallel* provides us with an *OR* function.
- » Connecting Switches in Serial provides us with an AND function.
- Now all we have to do is replace our switches with *MOSFET* transistors.

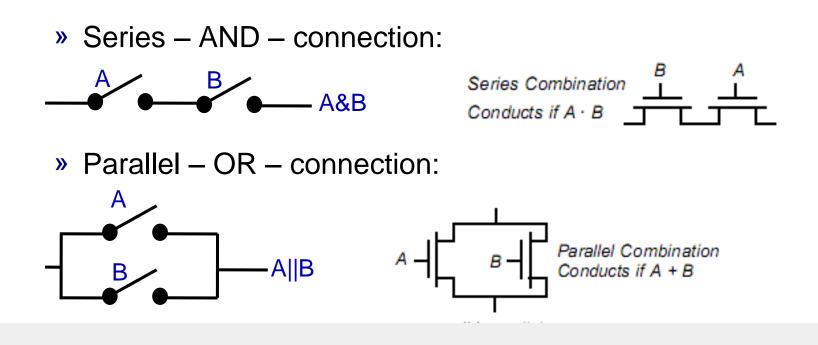






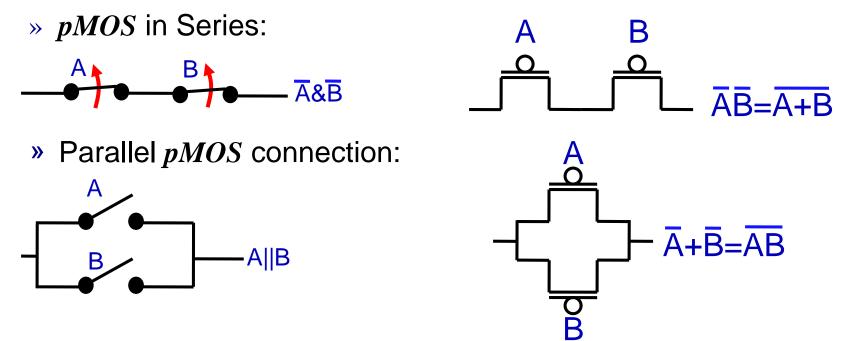
Series/Parallel Connection

□ Let's see what happens with *nMOS* transistors:





□ We need a *low input* to control *pMOS* transistors, so we get an *inverting* function:

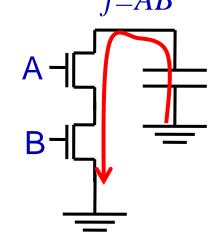


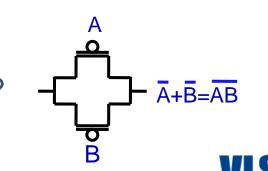
With *pMOS* transistors, we've created *NOR* and *NAND* functions!



- So we saw that we can easily create *inverting functions* (*NAND*, *NOR*) using *pMOS* transistors and *non-inverting functions* (*AND*, *OR*) using *nMOS*.
- □ Amazingly, this works out perfectly:
 - » When we implement a function in the *PDN*, it discharges the capacitance, or inverts the output.
 - » So if we use *nMOS* transistors in the *PDN*, we are getting *inverting functions*, just like the *pMOS* we'll use in the *PUN*!
- So can we get a NOR by using a pair of parallel nMOS in the PDN and a pair of parallel pMOS in the PUN?
 - » No, the parallel pair of *pMOS* give us a *NAND*, remember?...

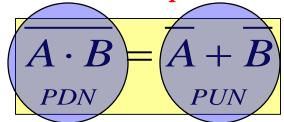






□ How then, do we realize our function properly?

- » You've probably figured out by now that a parallel pair of transistors in the *PDN* cooperates with a series pair in the *PUN*.
- » Let's look at the *2-input NAND*:



- » Discharging the load capacitance *inverts* the output of the *PDN*, giving us the left side of the equation.
- » The "*little circle*" on the pMOS inverts the inputs to the PUN, giving us the right side of the equation
- We've used complementary functions in the PUN and PDN to realize our function.

В

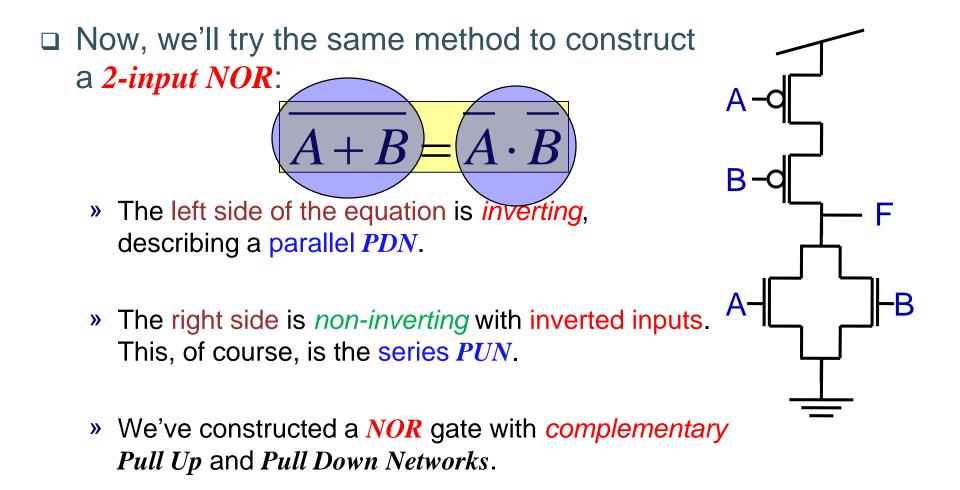
 V_{DD}

οF

Α	B	PUN	PDN	OUT
0	0			
0	1			
1	0			
1	1			



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Α	B	PUN	PDN	OUT
0	0			
0	1			
1	0			
1	1			



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- □ Looking at the *NAND* and *NOR* gates, we can reach a few conclusions about *CMOS Digital Logic*:
 - » We have constructed a Universal Set, meaning we can create any Boolean Function with CMOS.
 - » The complementary gate is naturally inverting, implementing only functions such as NAND, NOR and XNOR. To implement non-inverting functions (AND, OR, XOR, etc.) we must add an Inverter after the initial stage.
 - » The number of transistors required to implement an *N-input* logic gate is 2N.



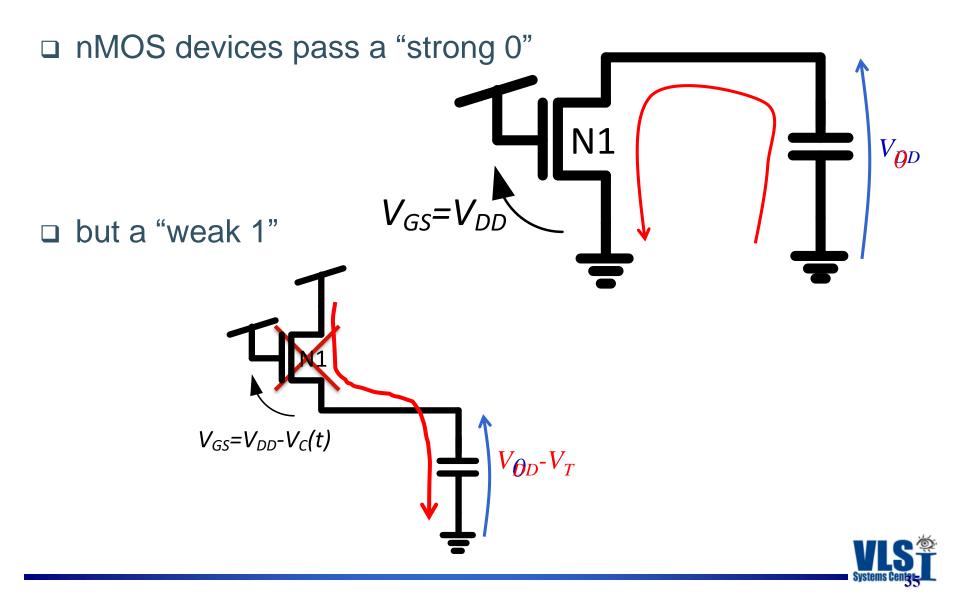
Last Week

Driving a load
CMOS Concept – NAND, NOR



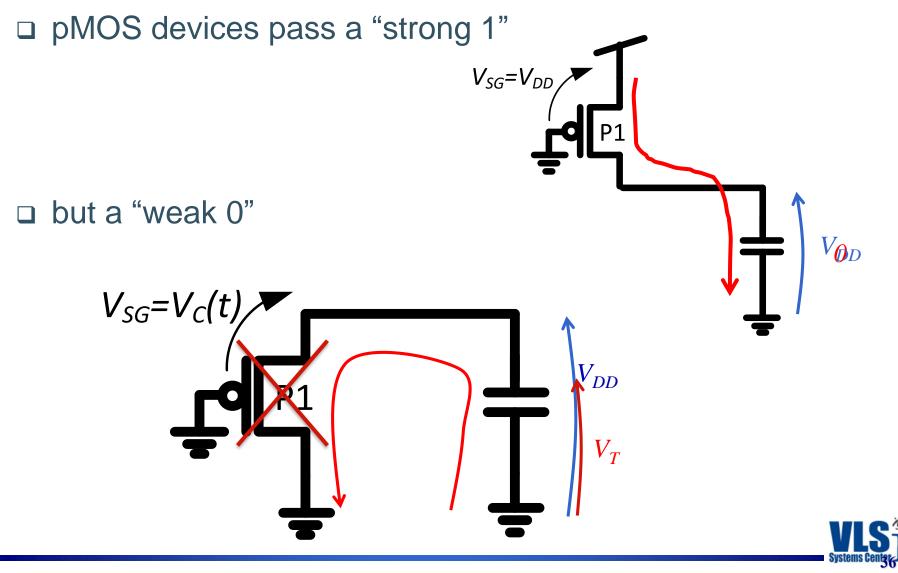
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How to choose nMOS/pMOS?



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How to choose nMOS/pMOS?



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How to choose nMOS/pMOS?

- □ Therefore, we will always:
- □ Implement the PDN with nMOS devices.
 - » These will create *inverting* functions.
- □ Implement the PUN with pMOS devices.
 - » These will create non-inverting functions.



Synthesis of a complex gate

Now let's try to implement a random Boolean Function using CMOS:

$$F = \overline{D + A \cdot (B + C)}$$

- » Our function is *inverting* with *uncomplemented* D *inputs*, so we can immediately derive the *PDN* from the given function.
- » **B** and **C** are *parallel*
- » A is in series with the **B+C** network
- » **D** is *parallel* to the A(B+C) network.

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B-

F

-C

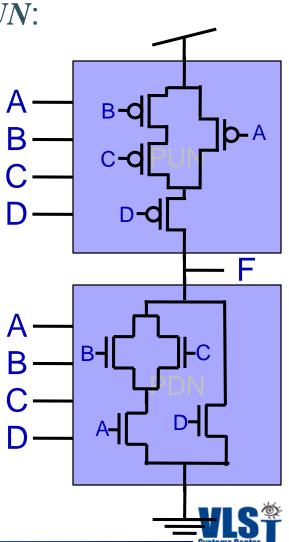
Α

Synthesis of a complex gate

□ Next we'll use *DeMorgan* to derive the *PUN*:

$$F = \overline{D + A \cdot (B + C)} = \overline{D} \cdot \overline{A \cdot (B + C)} =$$
$$= \overline{D} \cdot \left(\overline{A} + \overline{B + C}\right) = \overline{D} \cdot \left(\overline{A} + \overline{B} \cdot \overline{C}\right)$$

- » Our *PUN* function is *non-inverting* with complemented inputs.
- » **B** and **C** are in series
- » A is parallel with the BC network
- » **D** is in serial with the A + (BC) network.
- We could have also derived the *PUN* by marking subsets of the *PDN* and transforming them from *parallel* to *serial* and vice versa!



Another Example



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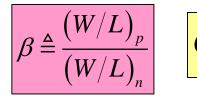


So we know about the topology, but to optimize the performance of the gate, let's take a look at

TRANSISTOR SIZING



- □ We previously developed an optimum sizing for a CMOS inverter based on the β ratio.
- □ We first marked the ratio between the PUN and PDN size as β and then expressed the load capacitance:



$$C_{load} = (1 + \beta) (C_{dn1} + C_{gn2}) + C_{wire}$$

We then expressed the delay and optimized it as a function of the resistance ratio between the PUN and PDN:

$$t_{pd} = \frac{0.69C_{load}}{2} \left(R_{eqn} + R_{eqp} \right) \qquad \frac{dt_{pd}}{d\beta} = 0 \qquad \beta_{opt} = \sqrt{\frac{R_{eqp}}{R_{eqn}}} \left(1 + \frac{C_{wire}}{C_{dn1} + C_{gn2}} \right) \approx \sqrt{\frac{R_{eqp}}{R_{eqn}}} \left(\frac{R_{eqp}}{R_{eqn}} \right)$$

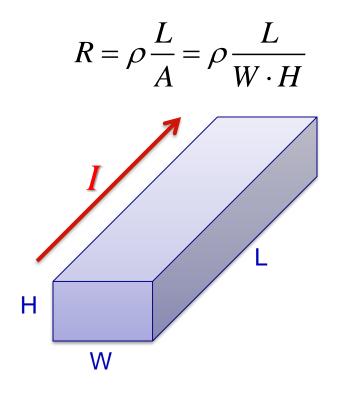
Transistor Sizing Methodology

- We will not prove it now, but it can be shown that the fastest (unloaded) CMOS gate is the optimal inverter.
- Therefore, our methodology for sizing an arbitrary CMOS gate is to try and make it have the same output resistance as the optimal inverter.
- □ To do this, we should look at how transistor *sizing* effects *resistance*.



Transistor Resistance

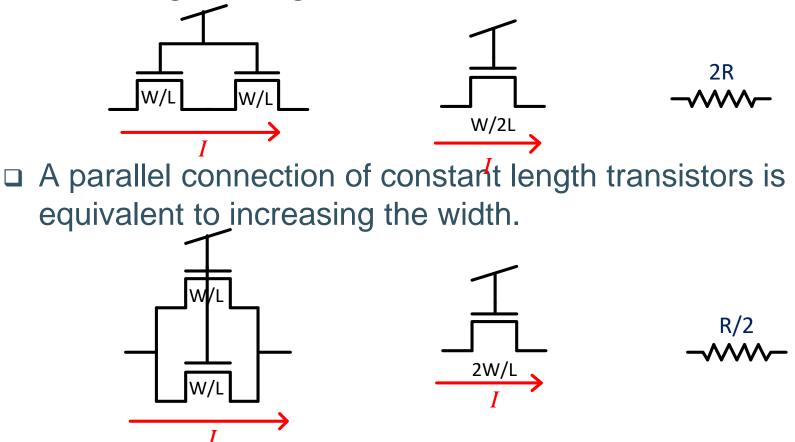
- We can intuitively look at a transistor's channel as a resistor.
- □ In physics we learned that resistance is given by:



 A transistor's channel is similar with a constant *H=channel depth* So *widening* a transistor will reduce the resistance, whereas a longer channel will increase the resistance.



Another look at transistor connections, shows that a series connection of constant width transistors is equivalent to increasing the length.



Transistor Resistance

□ This also can be shown according to our current model:

$$R_{eq} \approx \frac{3}{4} \frac{V_{DD}}{I_{DSAT} \left(1 + \lambda V\right)} \left(1 - \frac{7}{9} \lambda V_{DD}\right) \propto \frac{1}{\left(W/L\right)_n}$$

□ So to get equivalent resistance of a series connection:

$$R_{series} = R_{eq1} + R_{eq2} + \dots = const \left(\frac{1}{(W/L)_1} + \frac{1}{(W/L)_2} + \dots \right) = \frac{const}{(W/L)_{eq}} \left[\frac{1}{(W/L)_{eq}} = \frac{1}{(W/L)_1} + \frac{1}{(W/L)_2} + \dots \right]$$

□ And for a parallel connection:

$$(W/L)_{eq} = (W/L)_1 + (W/L)_2 + \dots$$

□ For example, take 2 transistors with W/L=4:

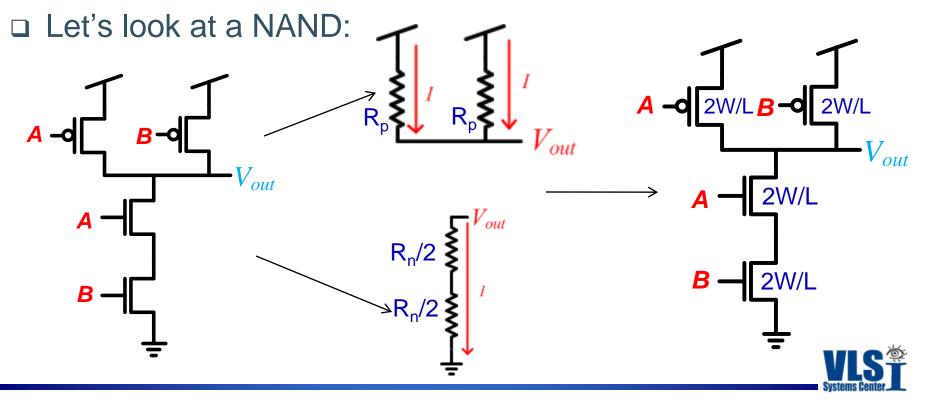
$$\frac{(W/L)_{parallel}}{=4+4=8} R_{eq, par} = \frac{1}{8} R_{min} \qquad \frac{(W/L)_{series}}{=\frac{1}{1/4+1/4}} = 2 R_{eq, ser} = \frac{2}{4} R_{min}$$

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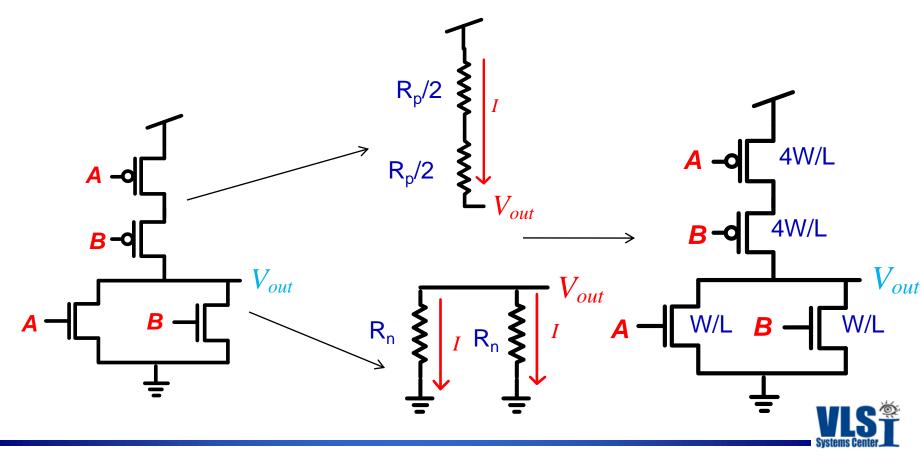
Transistor Sizing

- We'd like to now apply this to get the equivalent resistance of a gate to be similar to an optimum inverter.
- □ Since a complex gate has several paths, we will always take the *worst case* path for the PUN and PDN.



Transistor Sizing

□ What about a NOR?



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Conclusions

□ A short conclusion:

- » Minimizing the area of a transistor (WxL) is essential, as it affects:
 - Input Capacitance
 - Output Capacitance
 - Silicon area (cost)
 - Others
- » If we assume $W_{min} = L_{min}$ and calculate the areas of the *NAND* and *NOR* above, we will find:

$$A_{NAND} = 2(W_{eqp} + W_{eqn})L_{\min} = 2(2W_{\min} + 2W_{\min})L_{\min} = 8L_{\min}^{2}$$
$$A_{NOR} = 2(W_{eqp} + W_{eqn})L_{\min} = 2(4W_{\min} + W_{\min}) = 10L_{\min}^{2}$$

» So a NAND is much more efficient as a CMOS Logic Gate than a NOR!



Transistor Sizing

Another conclusion:

- » As we saw, each additional input requires 2 additional transistors, an nMOS and a pMOS.
- » This drastically increases both the *chip area* and the *capacitances* of a gate, increasing the *Propagation Delay*.
- » Using the *transistor sizing* technique, we can try to preserve the gate's *performance* at the expense of *size*, but this only works up to a limit.
- » A practical limit has been found to be a maximal Fan In of 4.
- » If a more complex function needs to be implemented, it should be done by cascading multiple stages of logic gates.

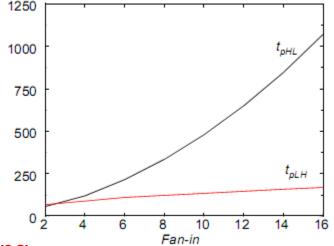


Figure 6.13 Propagation delay of CMOS NAND gate as a function of fan-in. A fan-out of one inverter is assumed, and all pull-down transistors are minimal size.







6.2 Circuit Implementation

6.3 Transistor Sizing

6.4 Dealing with High Fan-In

6.5 Non-Standard Gates

Okay, so we see that CMOS has a problem with high fan-in. So here are a few concepts in

DEALING WITH HIGH FAN-IN



Input Pattern Effects on Static Properties

□ How do we draw the VTC of a NAND gate?



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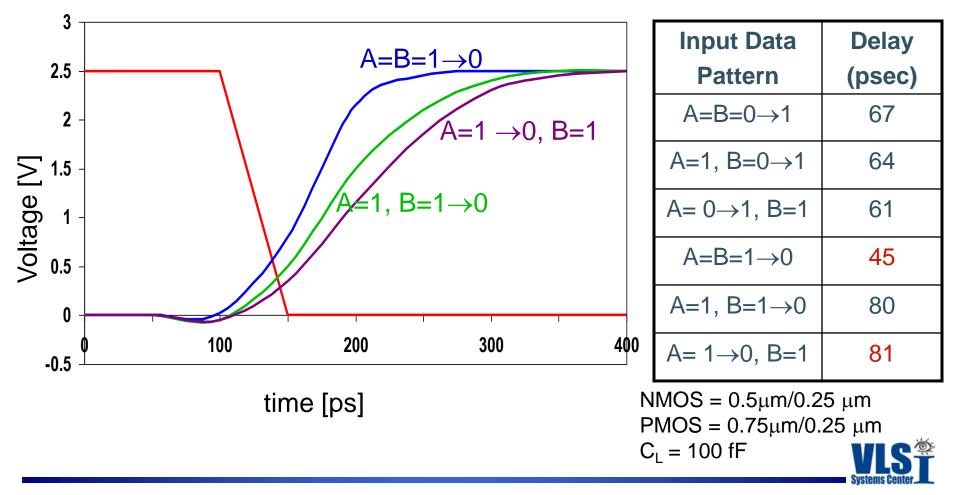
Input Pattern Effects on Delay

□ And what about the tpd of a NAND gate?



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Delay Dependence on Input Patterns



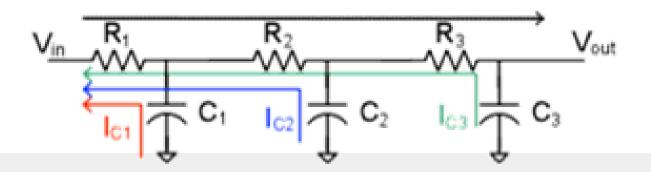
Delay Dependence on Input Patterns

Input Data Pattern	Delay (psec)	
A=B=0→1	67	
A=1, B=0→1	64	
A= 0→1, B=1	61	
A=B=1→0	45	
A=1, B=1→0	80	
A= 1→0, B=1	81	



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Delay Estimation using the Elmore Delay



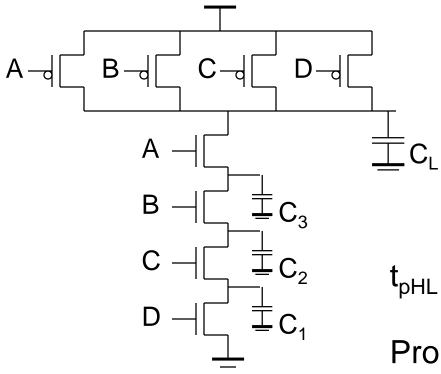


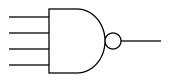


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Fan-In Considerations





Distributed RC model (Elmore delay)

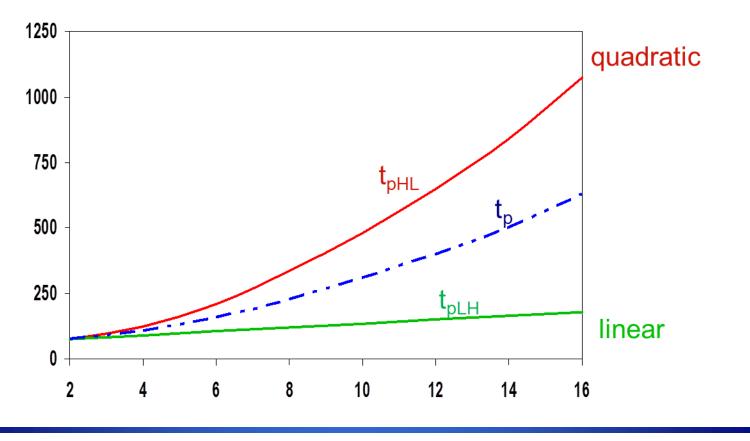
 $t_{pHL} = 0.69 R_{eqn}(C_1 + 2C_2 + 3C_3 + 4C_L)$

Propagation delay deteriorates rapidly as a function of fan-in – quadratically in the worst case.



Fan-In Considerations

CMOS Gates with a Fan In greater than 4 should be avoided!



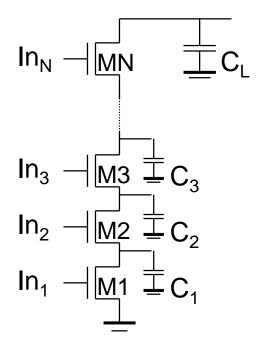
Dealing with Fan In: Transistor Sizing

- Make the transistors bigger, their resistance goes down, and the time constant decreases.
- BUT, the capacitance gets bigger presenting a bigger load to previous gates.
- □ (We will come back to this later...)



Dealing with Fan In: Progressive Sizing

Progressive sizing



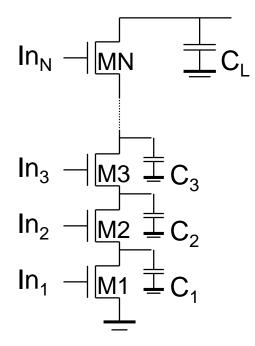
Looking at the Elmore Delay, M1 is on the path of all Capacitors, while MN is only on the path of CL.

Why not make M1 have less resistance than MN...



Dealing with Fan In: Progressive Sizing

Progressive sizing



M1 > M2 > M3 > ... > MN (the FET closest to the output is the smallest)

Can reduce delay by more than 20%; decreasing gains as technology shrinks

But it can have a large area overhead in layout...



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Fan In Considerations: Input Ordering

□ Not all logic paths are equal.

□ The frequency is measured according to the slowest path, or better known as the *critical path*.

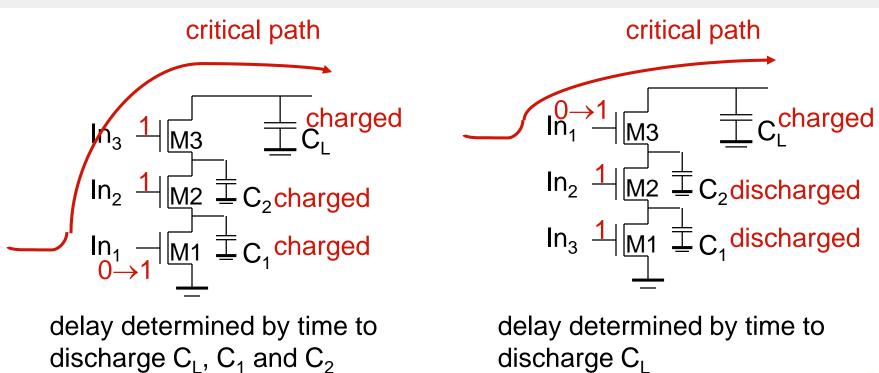
□ So we should connect the critical path to the faster inputs.



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Dealing with Fan In: Input Reordering

Transistor ordering





Dealing with Fan In: Input Reordering



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The VLSI Systems Center - BGU

Lecture 6: CMOS Digital Logic 65

Fan In Considerations: Logic Restructure

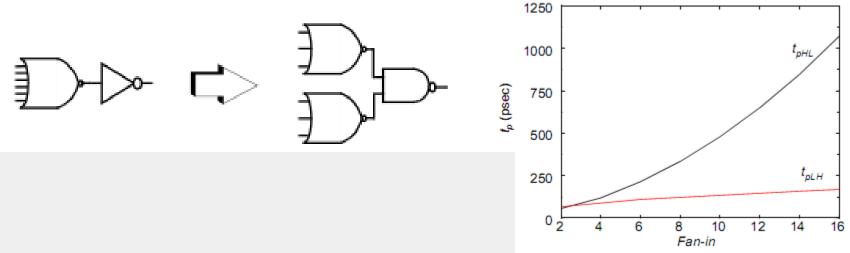
Sometimes we just have to have large Fan-Ins. For Example: A Decoder



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Dealing with Fan In: Logic Restructuring

We can usually restructure our logic (Boolean manipulations) to decrease the Fan-In of each gate by trading off number of stages with Fan In of each stage.



Next lecture, we will learn how to optimize this consideration.



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6.2 Circuit Implementation

6.3 Transistor Sizing

6.4 Dealing with High Fan-In

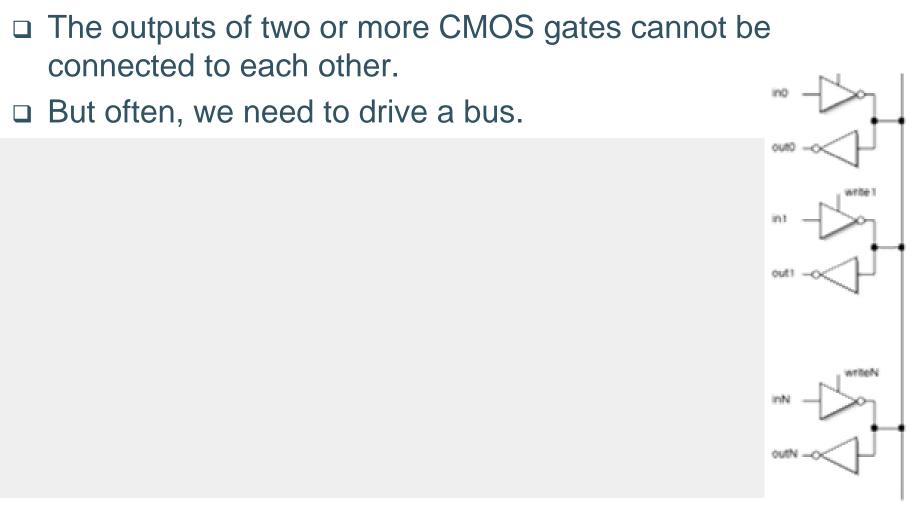
6.5 Non-Standard Gates

Okay, we now know how to synthesize any gate with the CMOS concept. Or do we? Here are some

NON-STANDARD GATES



Tri-State Buffers



□ Therefore, we need to implement a *tri-state* buffer.



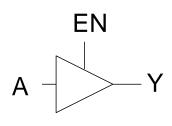
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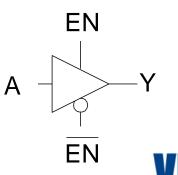
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Tri State Buffers

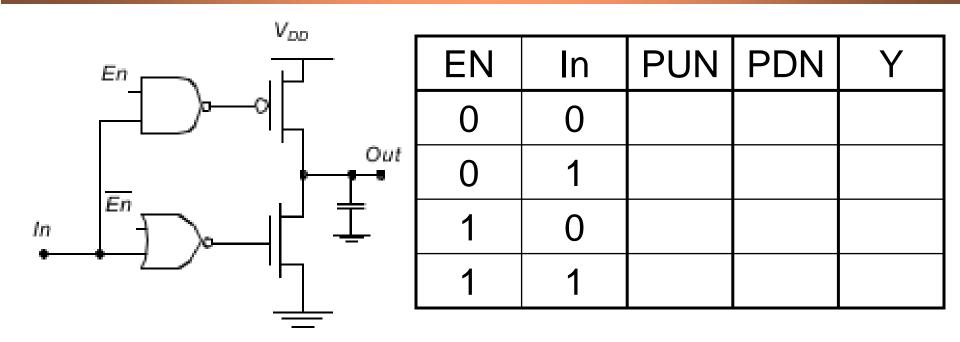
- Tri-state buffers are used when multiple circuits all connect to a common wire. Only one circuit at a time is allowed to drive the bus. All others can "disconnect" their outputs, but can "listen".
- □ Tri-state buffers enable "bidirectional" connections.





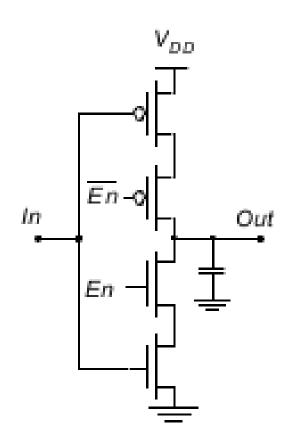


Standard CMOS Implementation





Reduced Transistor Implementation

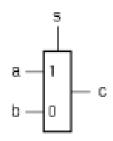


EN	In	PUN	PDN	Y
0	0			
0	1			
1	0			
1	1			

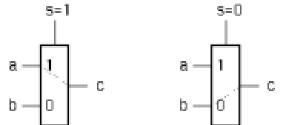


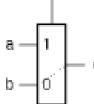
Tri-State Multiplexor

Multiplexor



If s=1 then c=a else



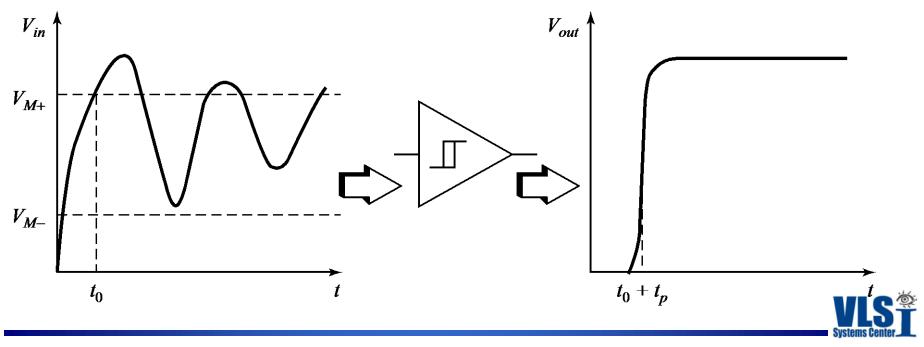




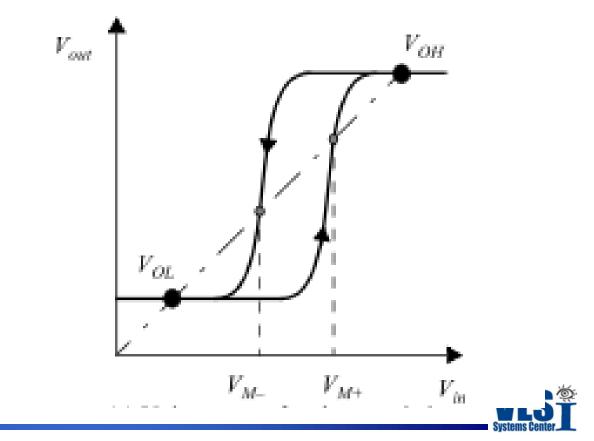
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Shmitt Trigger

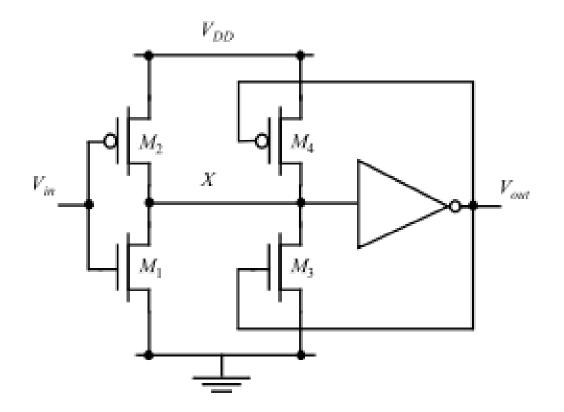
- Sometimes, we have a very noisy or slow varying signal, and would like to "clean it up".
- This is often the case in inter-chip interfaces, where there are many noise sources.



A Schmitt Trigger achieves our goal by using a *hysteresis* in its VTC:



Schmitt Trigger – CMOS Implementation

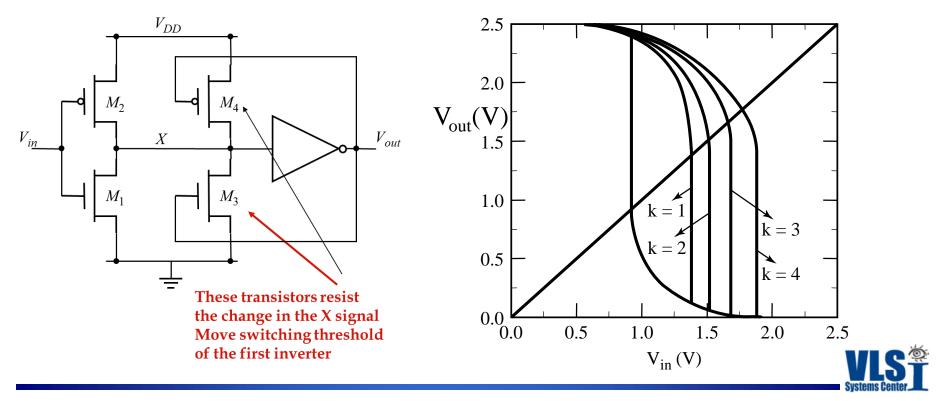




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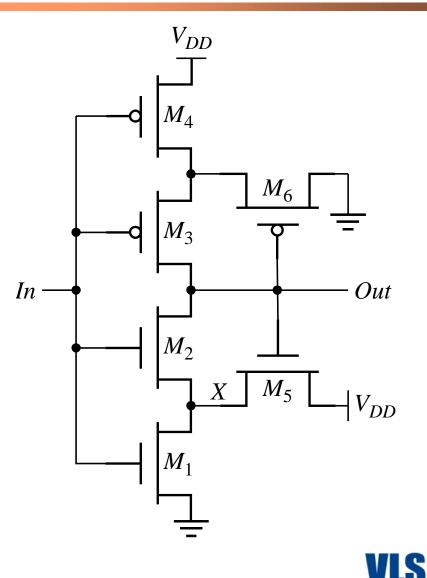
Schmitt Triggers

- □ Increasing kn/kp ratio decreases the logical switching threshold
- □ If V_{in} =0 then V_{out} (connected to M_4) is also zero
- □ So effectively the input is connected to M_2 and M_4 in parallel
- This increases kp and the switching threshold. If V_{in}=0 the situation is reversed and kn increases reducing the switching threshold



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Another Schmitt Trigger Implementation





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